Second Midterm Exam – Math 225

Fall 2013 –

Exercise 1:

Solve the given initial value problem and give the largest interval I over which the solution is defined:

$$\begin{cases} x\dfrac{dy}{dx} + 4y = x^3 - x \\ y(1) = 0 \end{cases}$$

Exercise 2:

Solve the given initial value problem

$$\begin{cases} (e^x + y)dx + (2 + x + ye^y)dy = 0 \\ y(0) = 1 \end{cases}$$

Exercise 3:

Solve the given initial value problem by finding an appropriate integrating factor

$$\begin{cases} xdx + (x^2 y + 4y)dy = 0 \\ y(4) = 0 \end{cases}$$

Exercise 4:

Solve the given differential equation by using an appropriate substitution

$$x^2 \frac{dy}{dx} - 2xy = 3y^4$$

Exercise 5:

Let $y_1(x) = x; y_2(x) = x^{-2}; y_3(x) = x^{-2}\ lnx$

a) Verify that $y_1, y_2\ and\ y_3$ form a fundamental set of solutions of the differential equation

$$x^3\, y''' + 6x^2\, y'' + 4x\, y' - 4y = 0 \qquad \text{On } (0, \infty)$$

b) Form the general solution.