NAME:

Group Number:

244

Second Midterm, April 2013

I) Choose the correct answer:

(a) Which of the following sets of vectors forms a basis of $\mathbb{R}^2$?

{(1, 1), (3, 1)}	{(2, 1), (1, -1), (0, 2)}	{(0, 1), (0, -3)}

(b) If $u = (-2, -3, 4, -6)$, $v = (4, 1, 6, 16)$ and $w = (8, -13, 0, 20)$, then the vector $x \in \mathbb{R}^4$ for which $5x - 2v + 3u = 2(w - 5x)$ is

x=(2, 1, 0, -6)	x=(2, -1, 0, 6)	x=(-2, -1, 0, 6)

(c) If $A = \begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix}$, then the set of matrices $\{A, B\}$ is linearly independent if

$B=\begin{bmatrix} 3 & 4 \\ -2 & 0 \end{bmatrix}$	$B=\begin{bmatrix} 3 & -4 \\ -2 & 0 \end{bmatrix}$	$B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(d) If $||\alpha(-2, 2, -1)|| = 3$, then the values that α can take are

{2, 3}	{-2, 2}	{-1, 1}

(e) If u and v are vectors in $\mathbb{R}^n$, then $\mathrm{d}(2u + v, u)$ equals

||u+v||	2||u||	||v||

II) Decide if the following statements are true (T) or false (F). Justify your answer.

(a) For all $u, v \in \mathbb{R}^n$, $||u+v|| = ||u|| + ||v||$.

T

F

(b) The coordinate vector $(P(X))_S$ of $P(X) = 2 - X + 3X^2$ with respect to the basis $S = \{1+X, 1-X, X^2\}$ of $\mathcal{P}_2(X)$ is $(2, -1, 3)$.

T

F

(c) The column vectors of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ are linearly independent.

T

F

(d) If $||u+v|| = 5$ and $||u-v|| = 3$, then $u \cdot v = 6$.

T

F

(e) Let $V = \mathbb{R}$. If the addition and multiplication on V are defined as $a + b = a^b$ and $k(a) = ka$, for all $a, b \in V$ and all scalars $k \in \mathbb{R}$, then V is a vector space.

T

F

III) Determine the value of a such that the matrices

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & -4 \\ a & -2 \end{bmatrix}$$

are linearly independent.

IV) Determine whether

$$V = \{(x, y) : x, y \in \mathbb{R}\}$$

is a vector space, when addition and multiplication on V are defined by

$$(x, y) + (x', y') = (xx', yy'), \quad \forall (x, y), (x', y') \in V$$

respectively

$$k(x, y) = (kx, ky), \quad \forall k \in \mathbb{R}, \forall (x, y) \in V.$$

V) Show that $\{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$, if $v_1 = (1, 1, 0)$, $v_2 = (1, 1, 1)$ and $v_3 = (0, 1, -1)$.

VI) Prove that $W = \{(a, b, c) \in \mathbb{R}^3 : b = 5a, c = 0\}$ is a subspace of $\mathbb{R}^3$.