King Saud University
Department of Mathematics
2nd Semester 1432-1433 H

MATH 244 (Linear Algebra)
2nd Midterm Exam
Duration: 90 Minutes

Student's Name	**Student's ID**	**Group No.**	**Lecturer's Name**

Question No.	**I**	**II**	**III**	**IV**	**Total**
Mark					

[**I**] Determine whether the following is **True** or **False**.

(1) If $S = \{\mathbf{u}, -\mathbf{v}\}$ is linearly independent , then so is $\{\mathbf{u}, \mathbf{v}\}$. ( )

(2) $W = \left\{(x, y) \in R^2 : y = 7x\right\}$ is a subspace of $\mathbb{R}^2$. ( )

(3) The functions $f_1(x) = 1,\ f_2(x) = e^x$ and $f_3(x) = xe^x$ are linearly dependent. ( )

(4) $S = \{(1, 1, 1), (0, 1, 2), (0, 2, 3)\}$ is a basis for $\mathbb{R}^3$. ( )

(5) If $\mathbf{u}, \mathbf{v} \in \mathbb{R}^2$ are such that $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\| \mathbf{u} \| = \| \mathbf{v} \| = 1$, then $d(\mathbf{u}, \mathbf{v}) = 0$. ( )

OVER

[II] Choose the correct answer.

(1) The vector (a, a, b) is a linear combination of the vectors $(0, 1, -1)$, $(1, -1, 0)$ if and only if

(a) $a = 2b$ **(b)** $b = 2a$ **(c)** $b = -2a$ **(d)** None of the previous.

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(2) The vectors $(1, 2, k)$ and $(3, k, 4)$ are orthognal if and only if k equals to

(a) $\frac{1}{2}$ **(b)** $\frac{-1}{2}$ **(c)** 2 **(d)** None of the previous.

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(3) If $A = \begin{bmatrix} 5 & 7 \\ 2 & -2 \end{bmatrix}$ then $(A)_S$, the coordinate vector of A relative to the basis

$$S = \left\{ \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \right\},$$

is

(a) $(1, -1, 2, 0)$ **(b)** $(1, -1, 0, 2)$ **(c)** $(1, 0, -1, 2)$ **(d)** None of the previous.

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(4) The dimension of the subspace $W = \left\{(a, b, c, d) \in \mathbb{R}^4 : a + 2b + 3c + d = 0\right\}$ is

(a) 2 **(b)** 3 **(c)** 4 **(d)** None of the previous.

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(5) If $u = (-2, 4, 1, 2) \in \mathbb{R}^4$, then $\| ku \| = 1$, for k equals

(a) 0 **(b) 1** **(c)** $\frac{-1}{5}$ **(d)** None of the previous.

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(6) Which of the following spans $\mathbb{R}^2$

(a) $\{(1, 1), (-1, -1)\}$ **(b)** $\{(1, 1), (-1, 0)\}$ **(c)** $\{(1, -1), (-1, 1)\}$ **(d)** $\{(0, 1), (0, -1)\}$

OVER

[**III**] Let $V = \mathbb{R}^2$ with the following addition and scalar multiplication operations on $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$:

$$\begin{aligned} \mathbf{u} + \mathbf{v} &= (u_1 + v_1, u_2 + v_2 - 1) \\ k\mathbf{u} &= (ku_1, ku_2) \end{aligned}$$

a- Find the object $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for all $\mathbf{u} \in V$.

b- If $\mathbf{u} \in V$. Find the object $\mathbf{v}$ such that $\mathbf{u} + \mathbf{v} = \mathbf{0}$

c- Show that V is not a vector space.

OVER

[IV] Find a basis and the dimension of the solution space for the following homogeneous linear system

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ x_2 - x_3 + x_4 &= 0 \\ x_1 - x_2 + x_4 &= 0 \end{aligned}$$

Good Luck