King Saud University
Department of Mathematics
1st Semester 1432

MATH 244 (Linear Algebra)
2nd Midterm Exam
Duration: 105 Minutes

Student's Name	**Student's ID**	**Group No.**	**Lecturer's Name**

Question No.	**I**	**II**	**III**	**IV**	**Total**
Mark					

[**I**] Determine whether the following is **True** or **False**. Justify your answer.

(1) The vector $\mathbf{b} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ is in the column space of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. (   )

(2) $S = \{(1,0,0), (0,2,0), (1,-2,0)\}$ spans R^3. (   )

(3) The solution space of the system $A\mathbf{x} = \mathbf{0}$ is a plane through the origin, where $A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3 \end{bmatrix}$. (   )

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(4) The coordinate vector $(p(x))_S$ of $p(x) = 2 - x + 3x^2$ with respect to the basis $S = \{1 + x, 1 - x, x^2\}$ of P_2 is $(2, -1, 3)$. ( )

(5) $W = \{A = [a_{ij}]_{n\times n} : A^T = -A\}$ is a subspace of M_{nn}. ( )

[II] Choose the correct answer.

(1) The vectors $\mathbf{v_1} = (0, 3, 1, -1)$, $\mathbf{v_2} = (6, 0, 5, 1)$ and $\mathbf{v_3} = (4, -7, 1, 3)$ are

(a) Linearly independent **(b)** Linearly dependent **(c)** None

(2) If $\mathbf{u} = (4, 1, 2, 2)$, $\mathbf{v} = (0, 2, 1, -2)$ and $\mathbf{w} = (3, 1, 2, 2)$ then $\|-2\mathbf{u}\| + \|2\mathbf{v}\|$ and $\left\|\frac{1}{\|\mathbf{w}\|}\mathbf{w}\right\|$ are respectively

(a) -4 and $3\sqrt{2}$ **(b)** -4 and 1 **(c)** 16 and 1 **(d)** None

(3) If V is a vector space with a basis $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$, then

(a) $dim(V) < 4$ **(b)** S is linearly dependent **(c)** $dim(V) = 4$ and $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ does not span V **(d)** None

(4) If $A = [a_{ij}]_{4\times 6}$ with $rank(A) = 2$, then

(a) $nullity(A^T) = 2$ **(b)** The number of parameters in the general solution of $A\mathbf{x} = \mathbf{b}$ is 3 **(c)** None

(5) If $\mathbf{f_1} = 1$, $\mathbf{f_2} = e^x$ and $\mathbf{f_3} = e^{2x}$, then $\{\mathbf{f_1}, \mathbf{f_2}, \mathbf{f_3}\}$

(a) Form a linearly independent set in $C^2(-\infty, \infty)$ **(b)** Form a linearly dependent set in $C^2(-\infty, \infty)$ **(c)** None

OVER

[III] For $A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix}$, find

(1) A basis for each of the row space of A, the column space of A, and the nullspace of A.

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(2) A basis of the row space of A consisting entirely of row vectors in A.

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[IV]

(1) Determine if the set of all real pairs (x, y) with operations $(x, y) + (x', y') = (xx', yy')$ and $k(x, y) = (kx, ky)$ form a vector space or not. Justify your answer.

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(2) Show that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$, where $\mathbf{v_1} = (2, 0, -1), \mathbf{v_2} = (4, 0, 7)$ and $\mathbf{v_3} = (-1, 1, 4)$, form a basis for R^3.

Good Luck