King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second-Mid Term Examination
(First Semester 1431/1432)

Max. Marks: 20 | Time: 90 minutes

**Marking Scheme: Q:1(3), Q:2(3), Q:3(3), Q:4(3), Q:5(4), Q:6(4)**

**Q. No: 1** Reverse the order of integration and evaluate the resulting integral $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\,dy\,dx$ .

**Q. No: 2** Evaluate the integral by changing to polar coordinates $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2y^2\,dx\,dy$ .

**Q. No: 3** Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the plane $z = 2$.

**Q. No: 4** A lamina having area mass density $\delta(x, y) = |x|$ at the point $P(x, y)$ and has the shape of the region bounded by the graphs of the equations $y = \sqrt{9 - x^2}$ , $y = 0$. Find the mass of the lamina.

**Q. No: 5** Evaluate the integral $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\,dz\,dy\,dx$ by changing it to **cylindrical coordinates.**

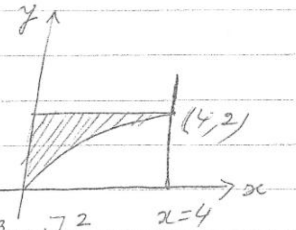
**Q. No: 6** Evaluate $\iiint_Q (x^2+y^2+z^2)\,dV$ , where $Q$ is the solid region that lies outside the sphere $x^2+y^2+z^2=1$ and inside the sphere $x^2+y^2+z^2=4$ by using **spherical coordinates.**

M-203

Max. Marks: 20    II Mid-term Exam. (I semester 1431/1432)

Q#1) Reverse the order of Integration and evaluate the resulting integral $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\,dy\,dx$.    [Mark: 3]

Soln. $\int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\,dx\,dy$    (2)




$= \int_0^2 \frac{1}{y^3+1} \left[x\right]_0^{y^2} dy$

$= \int_0^2 \frac{y^2}{y^3+1}\,dy = \frac{1}{3}\left[\ln(y^3+1)\right]_0^2$

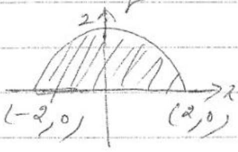
$= \frac{1}{3}\left[\ln 9 - \ln 1\right]$

$= \frac{1}{3}\ln 9$    (1)

Q#2) Evaluate the integral by changing to polar coordinates

$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2 y^2\,dx\,dy$    [Mark: 3]

Soln. $\int_0^{\pi} \int_0^2 r^4 \cos^2\theta \sin^2\theta \, r\,dr\,d\theta$    (2)




$= \int_0^{\pi} \left[\frac{r^6}{6}\right]_0^2 \cos^2\theta \sin^2\theta\,d\theta$

$= \frac{64}{6}\int_0^{\pi} (\cos\theta \sin\theta)^2\,d\theta = \frac{64}{6}\int_0^{\pi}\left(\frac{\sin 2\theta}{2}\right)^2 d\theta$

$= \frac{8}{3}\int_0^{\pi}\left(\frac{1-\cos 4\theta}{2}\right) d\theta = \frac{4}{3}\left\{\left[\theta\right]_0^{\pi} - \left[\frac{\sin 4\theta}{4\times 2}\right]_0^{\pi}\right\}$

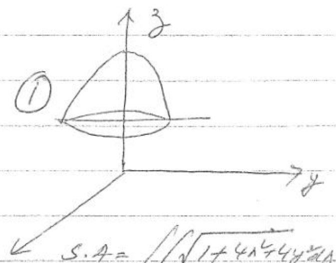
$= \frac{4\pi}{3}$    (1)

Q#3) Find the Surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the plane $z = 2$.

[Mark; 3]

Soln. $z = 4 - x^2 - y^2 = f(x, y)$

$f_x = -2x$ and $f_y = -2y$ (1)




we have $4 - x^2 - y^2 = 2$

or, $x^2 + y^2 = 2$

$\therefore$ $0 \le \theta \le 2\pi$

$0 \le r \le \sqrt{2}$

$S.A = \iint_R \sqrt{1 + 4x^2 + 4y^2}\, dA$

$\therefore$ Surface area $S = \int_0^{2\pi} \int_0^{\sqrt{2}} \sqrt{1 + 4r^2}\, r\, dr\, d\theta$ (1)

$= \int_0^{2\pi} \left[\frac{(1 + 4r^2)^{3/2}}{12}\right]_0^{\sqrt{2}} d\theta$ Put $1 + 4r^2 = t$, $8r\,dr = dt$

$= \frac{2\pi}{12}[27 - 1] = \frac{2\pi}{12}(26)$

$= \frac{13\pi}{3}$ (1)

Q#4) A lamina having area mass density $\delta(x, y) = |x|$ at the point $P(x, y)$ and has the shape of the region bounded by the graphs of the equations $y = \sqrt{9 - x^2}$, $y = 0$. Find the mass of the lamina. [Mark; 3]

Soln. Mass of the lamina: $m = \iint_R \delta\, dA = \int_{-3}^{3} \int_0^{\sqrt{9 - x^2}} |x|\, dy\, dx$

$= \int_{-3}^{3} |x| [y]_0^{\sqrt{9 - x^2}} dx = \int_{-3}^{3} x\sqrt{9 - x^2}\, dx$ (1)

$= \int_{-3}^{0} -x\sqrt{9 - x^2}\, dx + \int_0^3 x\sqrt{9 - x^2}\, dx$ (1)

$= \frac{1}{2}\left[\frac{(9 - x^2)^{3/2}}{3/2}\right]_{-3}^{0} - \frac{1}{2}\left[\frac{(9 - x^2)^{3/2}}{3/2}\right]_0^3 = \frac{1}{3}[(9)^{3/2}] - \frac{1}{3}[-(9)^{3/2}]$

$= \frac{27}{3} + \frac{27}{3}$

$= 9 + 9$

$= 18$ (1)

(3)

Q #5) Evaluate the Integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$ by changing it to cylindrical coordinates [4: Mark]

Soln. $\int_0^{2\pi}\int_0^2\int_r^2 r^2\, r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^2 (2-r)\,r^3\,dr\,d\theta$ (3)

$= \int_0^{2\pi}\left[2\frac{r^4}{4} - \frac{r^5}{5}\right]_0^2 d\theta$

$= 2\pi\left(8 - \frac{32}{5}\right) = \frac{16}{5}\pi$ (1)

Q #6) Evaluate $\iiint_Q (x^2+y^2+z^2)\,dV$, where $Q$ is the solid region that lies outside the sphere $x^2+y^2+z^2=1$ and inside the sphere $x^2+y^2+z^2=4$ by using spherical coordinates. [4: Mark]

Soln. $\int_0^{2\pi}\int_0^{\pi}\int_1^2 \rho^2\,\rho^2 \sin\varphi\,d\rho\,d\varphi\,d\theta$ (3)

$= \int_0^{2\pi}\int_0^{\pi}\sin\varphi\left[\frac{\rho^5}{5}\right]_1^2 d\varphi\,d\theta$

$= \frac{31}{5}\int_0^{2\pi}\int_0^{\pi}\sin\varphi\,d\varphi\,d\theta$

$= \frac{31}{5}\int_0^{2\pi}\left[-\cos\varphi\right]_0^{\pi} d\theta$

$= \frac{31}{5}\times 2\,[2\pi] = \frac{124}{5}\pi$ (1)