Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS
MATH 1110

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Reciprocal Identities

- This relationship can be summarized:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

- Each identity is true for angles except those that that make a denominator equal to zero
- These reciprocal identities must be memorized

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1,$$

$$\tan^2\theta + 1 = \sec^2\theta,$$

$$1 + \cot^2\theta = \csc^2\theta$$

MUST MEMORIZE!!!

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1-2\sin^2\theta = 2\cos^2\theta - 1$$

$$\tan 2\theta = 2\frac{\tan\theta}{1-\tan^2\theta}$$

Half Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

Angle Addition Formulas

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$$

Sum, Difference and Product of Trigonometric Functions

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Trigonometric Integrals

Recall Basic Identities

- Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

- Half-Angle Formulas

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

These will be used to integrate powers of sin and cos

Integral of $\sin^n x$, *n* Odd

- Split into product of an even and sin x

$$\int \sin^5 x\, dx = \int \sin^4 x \cdot \sin x\, dx$$

- Make the even power a power of $\sin^2 x$

$$\int \sin^4 x \cdot \sin x\, dx = \int \left(\sin^2 x\right)^2 \sin x\, dx$$

- Use the Pythagorean identity

$$\int \left(\sin^2 x\right)^2 \sin x\, dx = \int \left(1 - \cos^2 x\right)^2 \sin x\, dx$$

- Let u = cos x, du = -sin x dx

$$-\int \left(1 - u^2\right)^2 du = -\int 1 - 2u^2 + u^4\ du = \ldots$$

Integral of $\sin^n x$, *n* Odd

- Integrate and un-substitute

$$-\int 1-2u^2+u^4 \; du = -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + C$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 + C$$

- Similar strategy with $\cos^n x$, *n* odd