Chapter 2

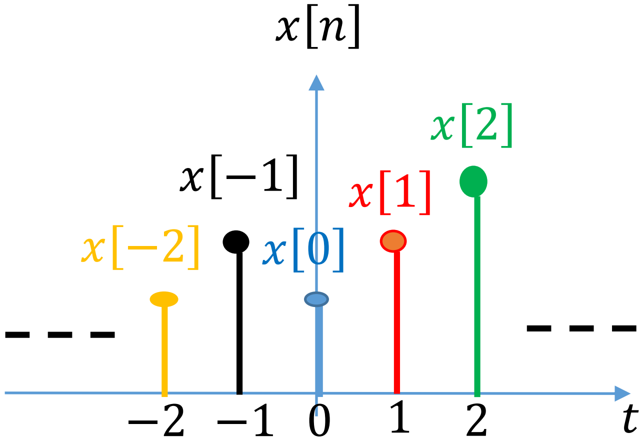
Linear Time-Invariant Systems

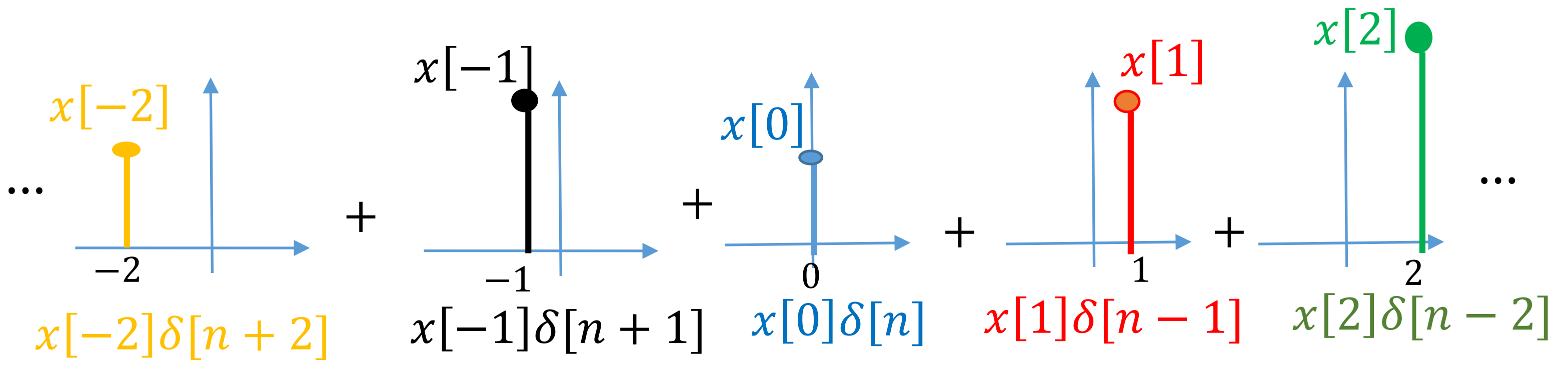
(LTI Systems)

Linear Time-Invariant Systems

- A system is said to be Linear Time-Invariant (*LTI*) if it possesses the basic system properties of *linearity* and *time-invariance.*
- The input-output relationship for LTI systems is described in terms of a *convolution* operation.

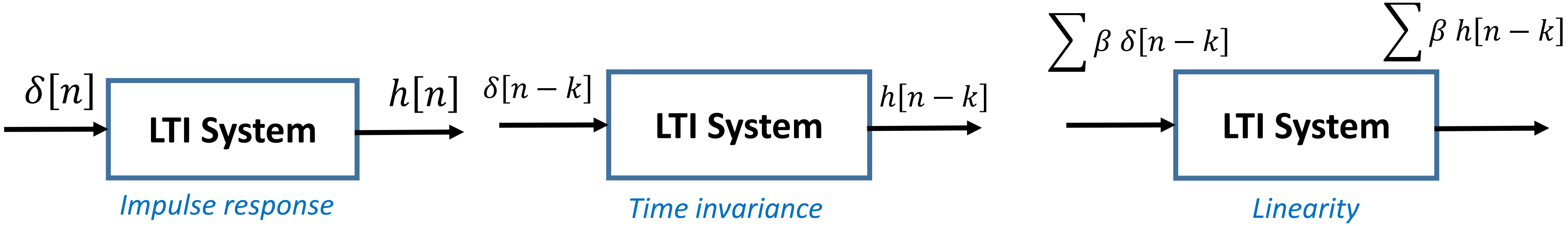
DT Signal Decomposition in terms of shifted unit impulses





$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Sum of scaled impulses



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

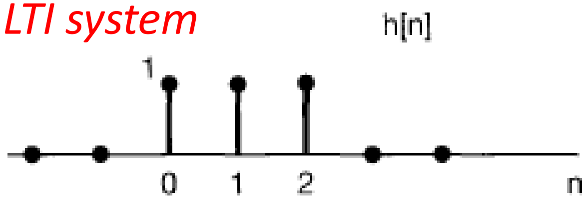
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Convolution sum

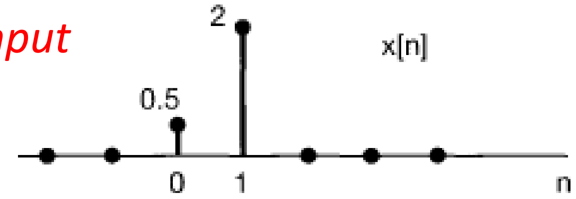
Convolution 1

Example:

Impulse response of an LTI system

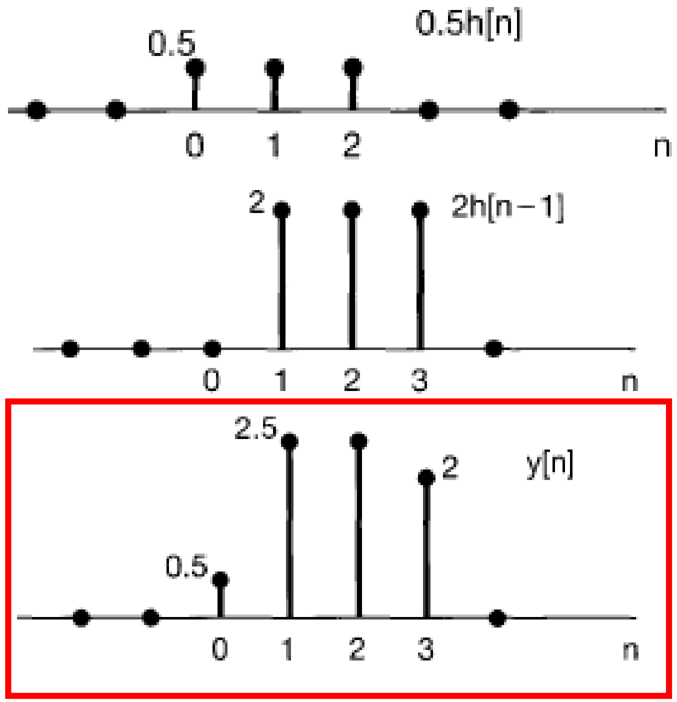


input



Find output

Convolution



$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

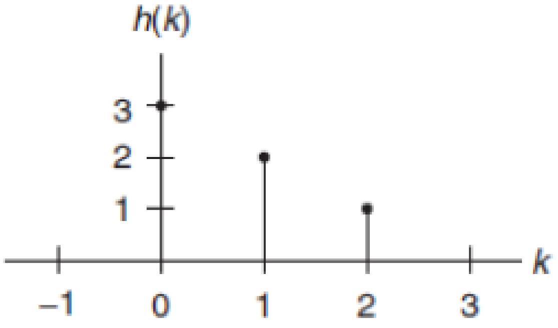
There are only two non-zero values for the input

$$y[n] = x[0]h[n-0] + x[1]h[n-1] = 0.5h[n] + 2\,h[n-1]$$

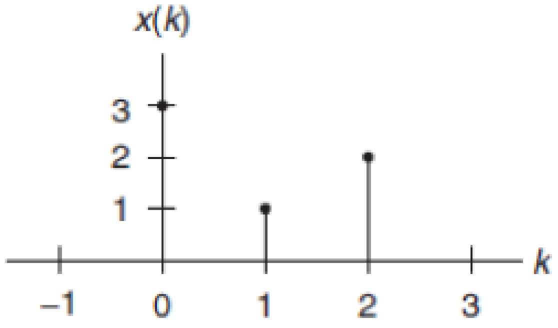
Convolution 2

Example:

$$x[n] = \begin{cases} 3 & n = 0 \\ 2 & n = 1 \\ 1 & n = 2 \\ 0 & elsewhere \end{cases}$$



Length =3



Length =3

$$h[n] = \begin{cases} 3 & n = 0 \\ 1 & n = 1 \\ 2 & n = 2 \\ 0 & elsewhere \end{cases}$$

Solution:

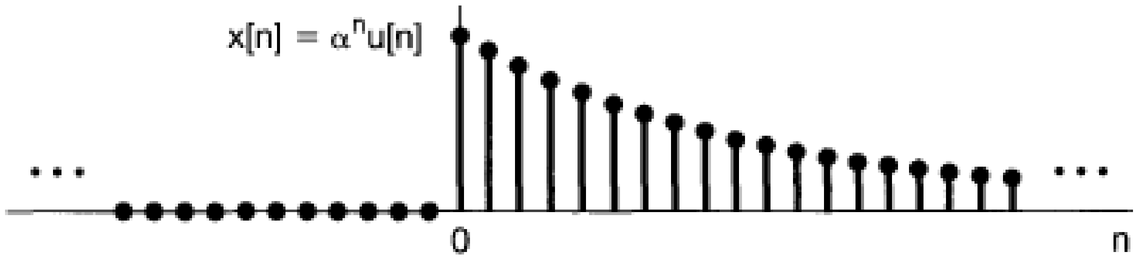
Convolution sum using the table method.

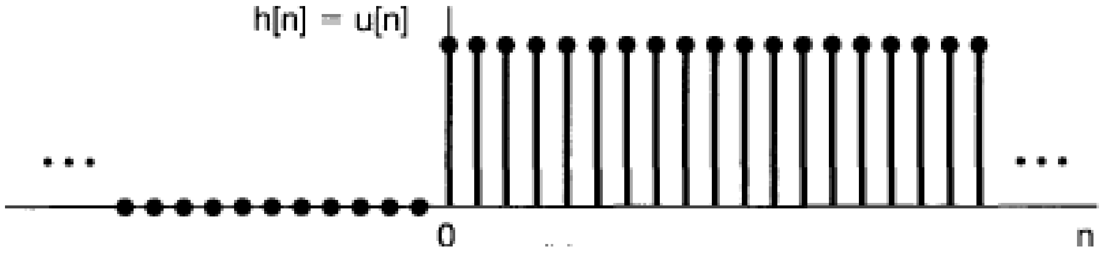
k:	-2	-1	0	1	2	3	4	5	
$x(k)$:			**3**	**1**	**2**				
$h(-k)$:	1	2	3						$y(0) = 3 \times 3 = 9$
$h(1-k)$		1	2	3					$y(1) = 3 \times 2 + 1 \times 3 = 9$
$h(2-k)$			1	2	3				$y(2) = 3 \times 1 + 1 \times 2 + 2 \times 3 = 11$
$h(3-k)$				1	2	3			$y(3) = 1 \times 1 + 2 \times 2 = 5$
$h(4-k)$					1	2	3		$y(4) = 2 \times 1 = 2$
$h(5-k)$						1	2	3	$y(5) = 0$ (no overlap)

Convolution Length = $N_1 + N_2 - 1 = 3 + 3 - 1 = 5$

Convolution 3

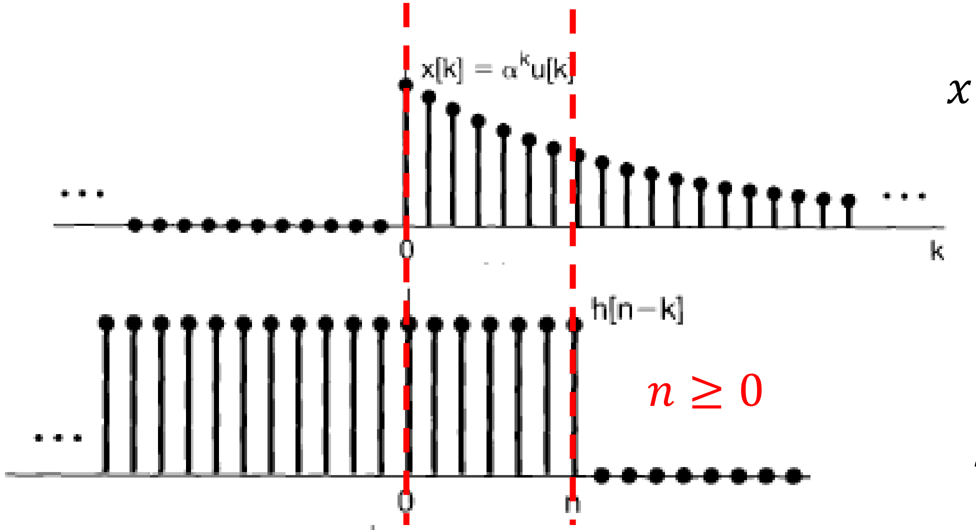
Example: *Find the output of an LTI system having a* unit impulse response $h[n]=u[n]$, *for the input* $x[n]=\propto^{n} u[n]$





$$x[k]h[n-k]=\begin{cases}\propto^{k} & 0\le k\le n\\ 0 & otherwise\end{cases}$$

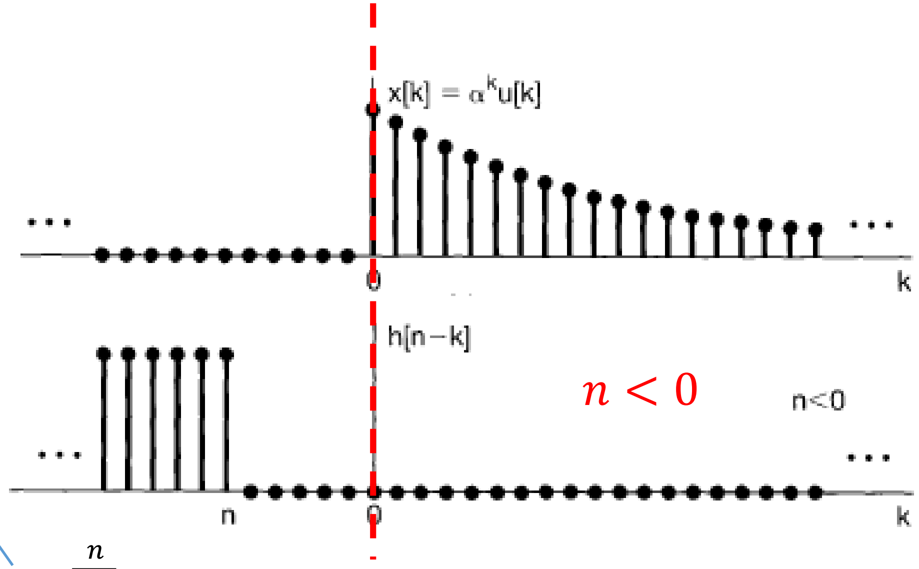
$k>n \;\rightarrow h[n-k]=0$
$k<0 \;\rightarrow x[k]=0$



$n\ge 0$

Thus, for $n\ge 0$,

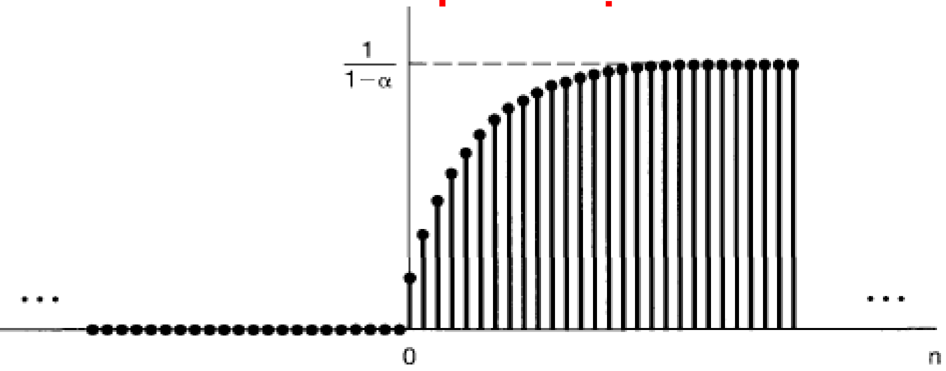
$$y[n]=\sum_{k=0}^{n}\propto^{k}=\frac{1-\propto^{n+1}}{1-\propto}$$



$n<0$

Thus, for all n,

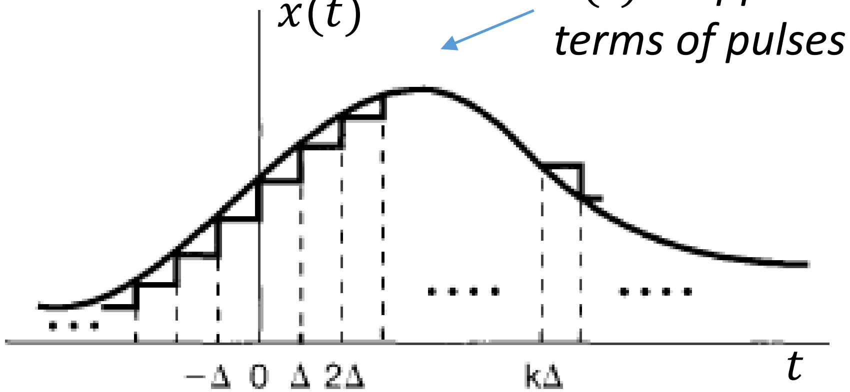
$$y[n]=\left(\frac{1-\propto^{n+1}}{1-\propto}\right)u[n]$$



$$s=\sum_{k=0}^{n}\propto^{k}=1+\propto+\propto^{2}+\cdots+\propto^{n}$$

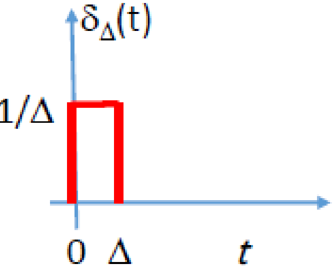
$$\propto s=\propto+\propto^{2}+\cdots+\propto^{n+1} \Rightarrow s-\propto s=1-\propto^{n+1}$$

The Representation of Continuous-Time Signals in Terms of Impulses



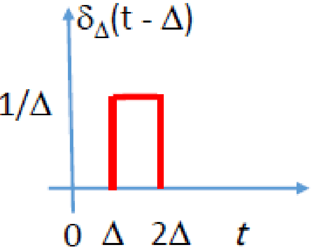
$x(t)$ is approximated in terms of pulses or staircase

$\hat{x}\,(t)$: *the approximation of* $x(t)$



At $t = 0$

$$\hat{x}\,(0) = x(0)\Delta\delta_\Delta(t) = \begin{cases} x(0) & 0 \le t \le \Delta \\ 0 & otherwise \end{cases}$$



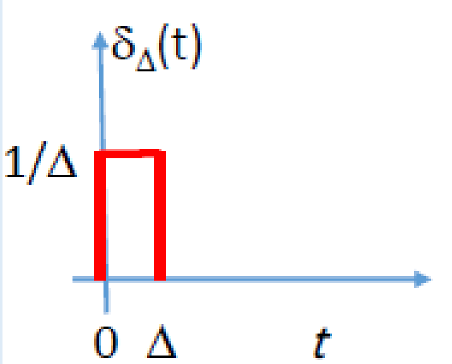
At $t = \Delta$

$$\hat{x}\,(\Delta) = x(\Delta)\Delta\delta_\Delta(t-\Delta) = \begin{cases} x(\Delta) & \Delta \le t \le 2\Delta \\ 0 & otherwise \end{cases}$$

In general *At* $t = k\Delta$

$$\hat{x}\,(k\Delta) = x(k\Delta)\Delta\delta_\Delta(t-k\Delta) = \begin{cases} x(k\Delta) & k\Delta \le t \le (k+1)\Delta \\ 0 & otherwise \end{cases}$$

Defining:



$$\delta_\Delta(t) = \begin{cases} 1/\Delta & 0 \le t \le \Delta \\ 0 & otherwise \end{cases}$$

$$\Delta\delta_\Delta(t) = \begin{cases} 1 & 0 \le t \le \Delta \\ 0 & otherwise \end{cases}$$

$\Delta\delta_\Delta(t)$ *has a unit amplitude*

The complete pulse/staircase approximation of $x(t)$ is the sum

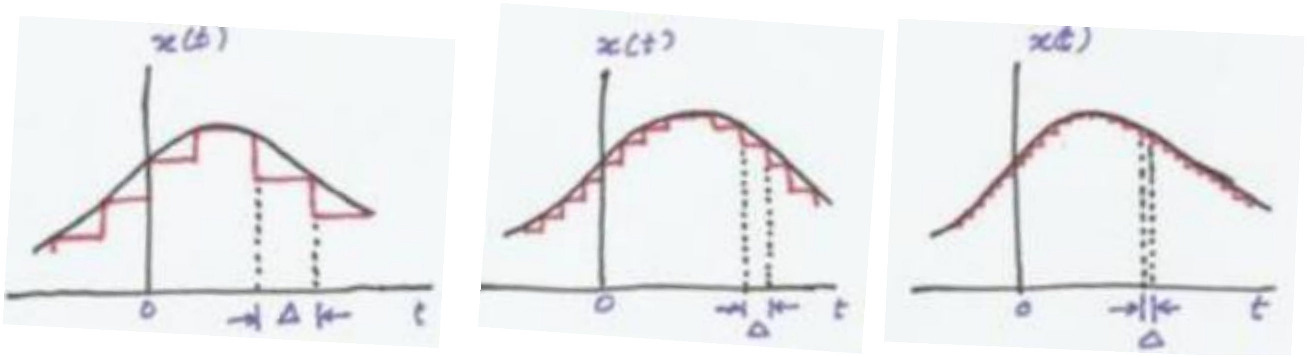
$$\hat{x}\,(t) = \cdots + \hat{x}\,(-\Delta) + \hat{x}\,(0) + \hat{x}\,(\Delta) + \cdots$$

$$\hat{x}\,(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

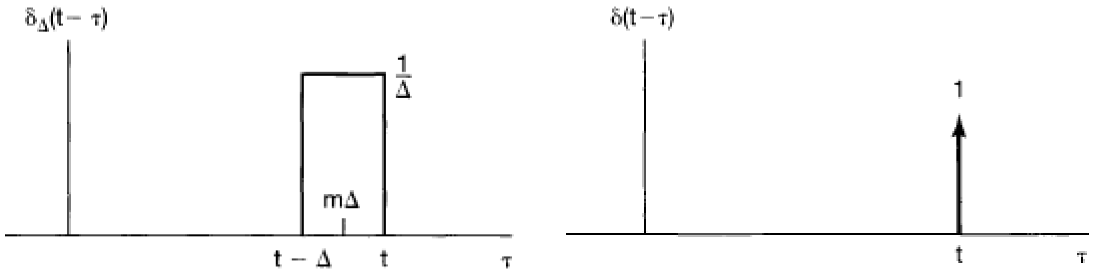
The Representation of CT Signals in Terms of Impulses$_2$

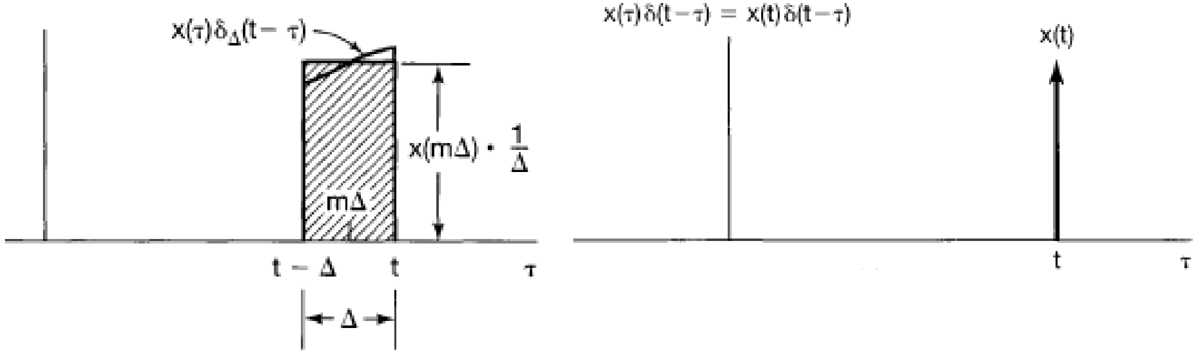
If we let Δ approach 0 $\hat{x}(t)$ *becomes closer and closer and equals* $x(t)$ *in the limit of* 0

$$x(t) = \lim_{\Delta\to 0} \hat{x}(t) = \lim_{\Delta\to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t - k\Delta)\Delta$$



In the limiting case the sum approaches integral (area):





$$\Delta \to 0;\ \delta_\Delta(t) \to \delta(t)$$

$$\sum_{k=-\infty}^{\infty} \cdots \Delta \to \int_{\tau=-\infty}^{\infty} \cdots\, d\tau$$

Consequently:

$$x(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

The Convolution Integral

- *The impulse response $h(t)$ of a continuous-time LTI system S*

$$h(t) = S\{\delta(t)\}$$

$$\delta(t) \longrightarrow \boxed{\textbf{CT LTI System}} \longrightarrow h(t)$$

Sum (Integral) of weighted shifted impulses

For the input $x(t)$:

$$y(t) = S\{x(t)\} = S\left\{\int_{\tau=-\infty}^{\infty} \underbrace{x(\tau)}_{\text{weight}}\, \underbrace{\delta(t-\tau)}_{\text{impulse}}\, d\tau\right\} \overset{\text{linearity}}{=} \int_{\tau=-\infty}^{\infty} x(\tau)\, S\{\delta(t-\tau)\}\, d\tau$$

- *Since the system is time-invariant:*

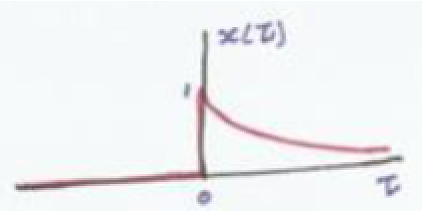
$$S\{\delta(t-\tau)\} = h(t-\tau)$$

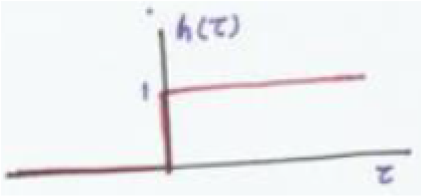
Time-invariance

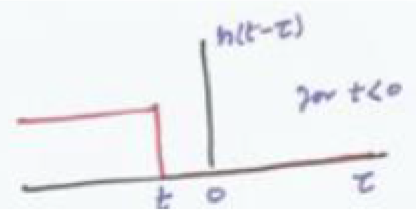
$$\boxed{y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t)}$$

Example 1

Let, the input x(t) to an LTI system with unit impulse response $h(t)$ be given as $x(t) = e^{-at}u(t)$ for $a > 0$ and $h(t) = u(t)$. Find the output $y(t)$ of the system.





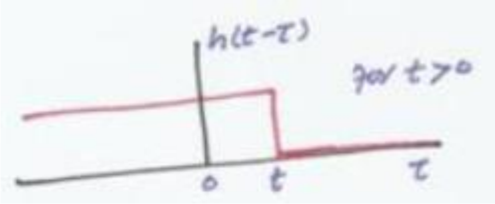


$$y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(t) \neq 0 \; for \; t \geq 0$

$h(t) = u(t).$

$$h(t-\tau) = \begin{cases} 1 & 0 < \tau < t \\ 0 & \tau > t \end{cases}$$

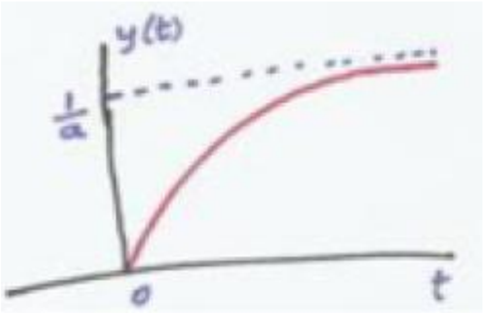


$$= \int_0^{\infty} e^{-a\tau}\, h(t-\tau)\, d\tau \;\; for \; t > 0$$

$$= \int_0^{t} e^{-a\tau}\, d\tau = \frac{1}{-a} e^{-a\tau}\Big|_0^t = \frac{1}{a}\left(1 - e^{-at}\right)$$

Thus, for all t, we can write

$$\boxed{y(t) = \frac{1}{a}\left(1 - e^{-at}\right)}$$



The Convolution Integral$_2$

Example 2

Find $y(t) = x(t) * h(t)$ *, where* $\left| \begin{array}{l} x(t) = e^{2t} u(-t) \\ h(t) = u(t-3) \end{array} \right.$

The system response is $y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

these two signals have regions of nonzero overlap

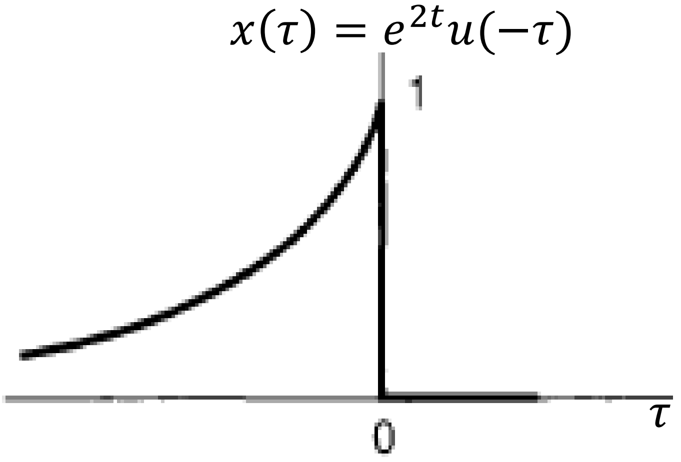
For $t-3 \le 0$: *nonzero overlap for* $-\infty \le \tau \le t-3$

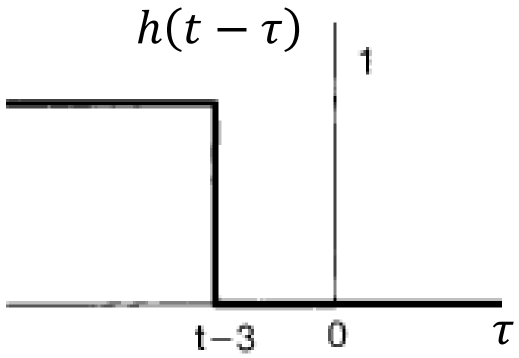
$$y(t) = \int_{-\infty}^{t-3} e^{2\tau}\, d\tau = \frac{1}{2} e^{2(t-3)} \qquad \textit{For } t \le 3$$

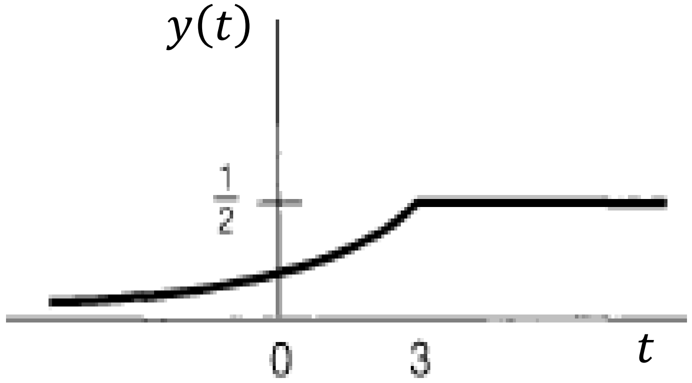
For $t-3 \ge 0$: *nonzero overlap for* $-\infty \le \tau \le 0$

$$y(t) = \int_{-\infty}^{0} e^{2\tau}\, d\tau = \frac{1}{2} \qquad \textit{For } t \ge 3$$

$$y(t) = \begin{cases} \frac{1}{2} e^{2(t-3)} & \textit{For } t \le 3 \\ 1/2 & \textit{For } t \ge 3 \end{cases}$$

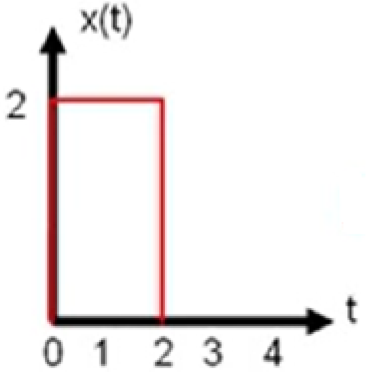


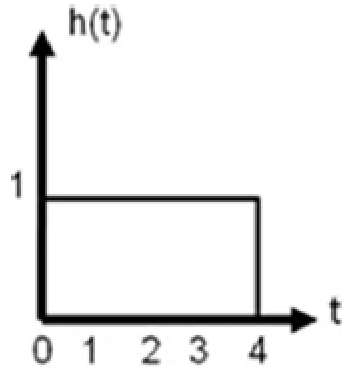




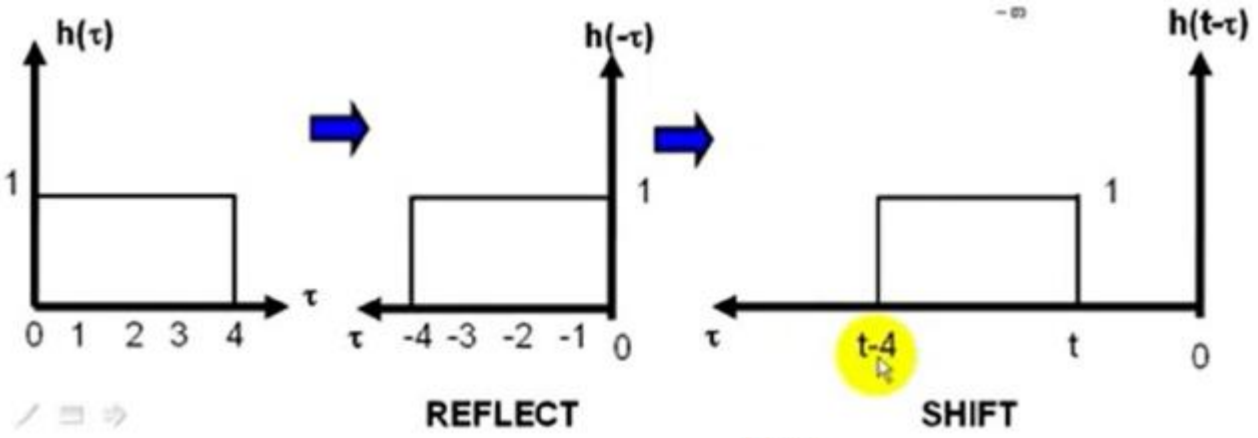
The Convolution Integral$_3$

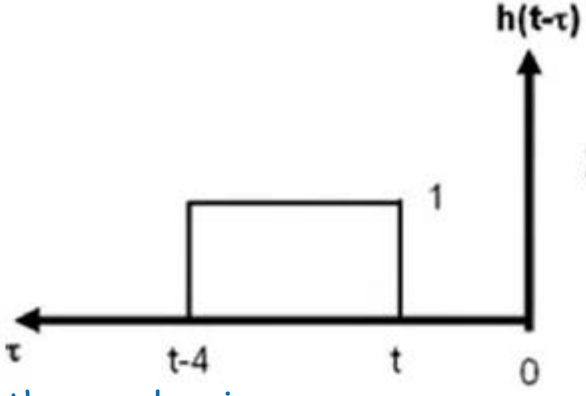
$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$





Convolution





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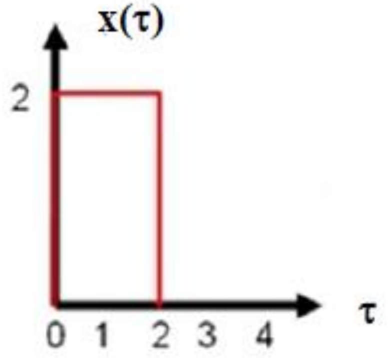
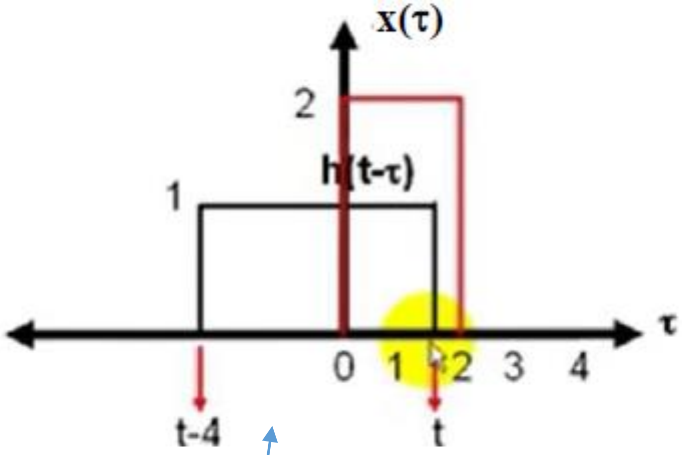


Shift $h(t-\tau)$



Aggregate the overlapping

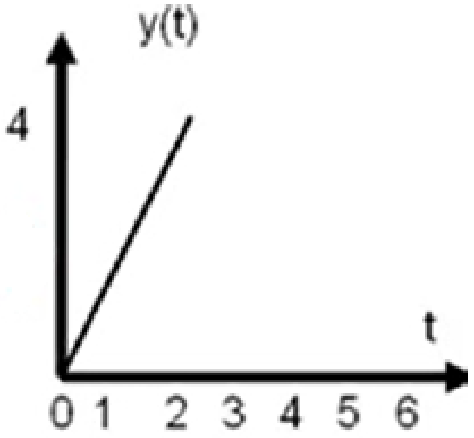
$\boldsymbol{t < 0}$ No overlapping $\quad y(t) = 0$

$\boldsymbol{0 < t < 2}$

$$y(t) = \int_0^t 2 \cdot 1\, d\tau \qquad 0 < \tau < t$$

$$y(t) = 2\left(\tau\Big|_0^t\right)$$

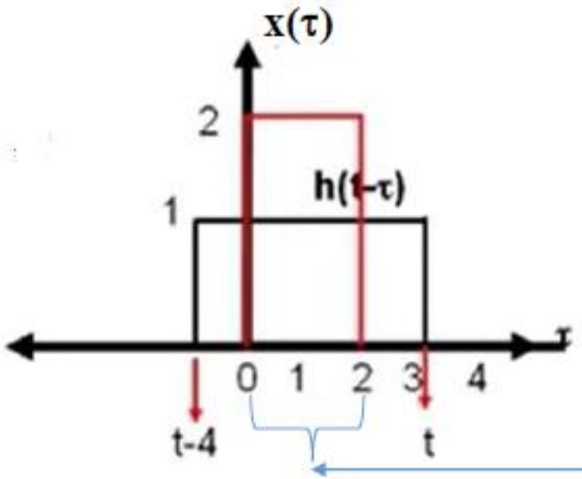
$$y(t) = 2t$$



$\boldsymbol{0 < t < 2}$

The Convolution Integral$_4$

$2 < t < 4$



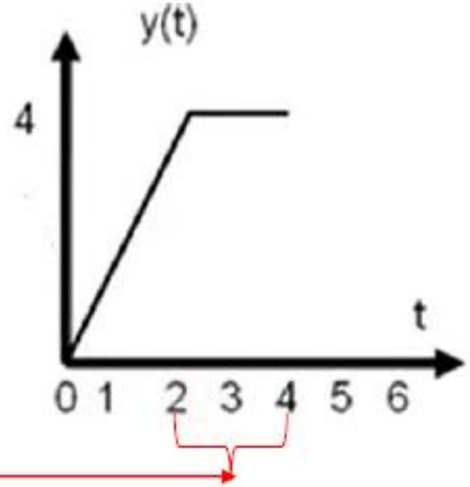
$t - 4 < 0 \qquad t > 2$

$\boxed{2 < t < 4}$

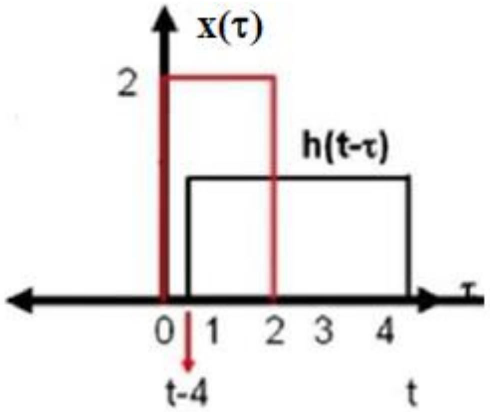
$\boxed{0 < \tau < 2}$

$$y(t) = \int_0^2 2 d\tau$$

$$y(t) = 4$$



$4 < t < 6$



$$y(t) = \int_{t-4}^{2} 2 \cdot 1 d\tau$$

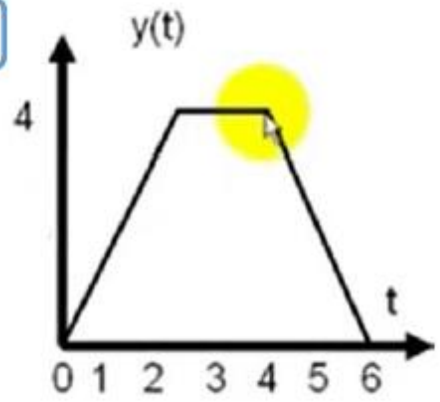
$\boxed{t - 4 < \tau < 2}$

$\boxed{t < 6}$

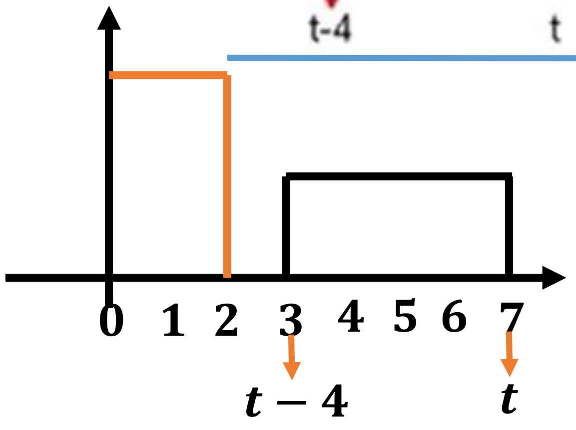
$$y(t) = 2 \cdot (\tau|_{t-4}^{2})$$

$$y(t) = 2(2 - (t - 4))$$

$$y(t) = -2t + 12$$



$t > 6$



No-overlap $\quad y(t) = 0$

Properties of LTI Systems

- *The characteristics of an LTI system are completely determined by its impulse response. This property holds in general only for LTI systems only.*
- *The unit impulse response of a nonlinear system does not completely characterize the behavior of the system.*

Consider a discrete-time system with unit impulse response:

$$h[n] = \begin{cases} 1, & n = 0,1 \\ 0, & otherwise \end{cases}$$

If the system is LTI, we get the system output (by convolution): $y[n] = x[n] + x[n-1]$

There is only one such LTI system for the given h[n].

However, there are many nonlinear systems with the same response, h[n].

Two different Non-Linear systems with same impulse response

$$y[n] = (x[n] + x[n-1])^2 \longrightarrow h[n] = (\delta[n] + \delta[n-1])^2 \longrightarrow h[n] = \begin{cases} 1, & n = 0,1 \\ 0, & otherwise \end{cases}$$

$$y[n] = \max(x[n], x[n-1]) \longrightarrow h[n] = Max(\delta[n], \delta[n-1]) \longrightarrow h[n] = \begin{cases} 1, & n = 0,1 \\ 0, & otherwise \end{cases}$$

1 Commutative Property

$$x(t) * h(t) = h(t) * x(t)$$

$$x[n] * h[n] = h[n] * x[n]$$

Proof: (discrete domain)

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Put $r = n - k \Rightarrow \quad k = n - r$

$$x[n] * h[n] = \sum_{r=-\infty}^{\infty} x[n-r]h[r] = \sum_{r=-\infty}^{\infty} h[r]x[n-r] = h[n] * x[n]$$

Similarly, we can prove it for continuous domain.

2 Distributive Property

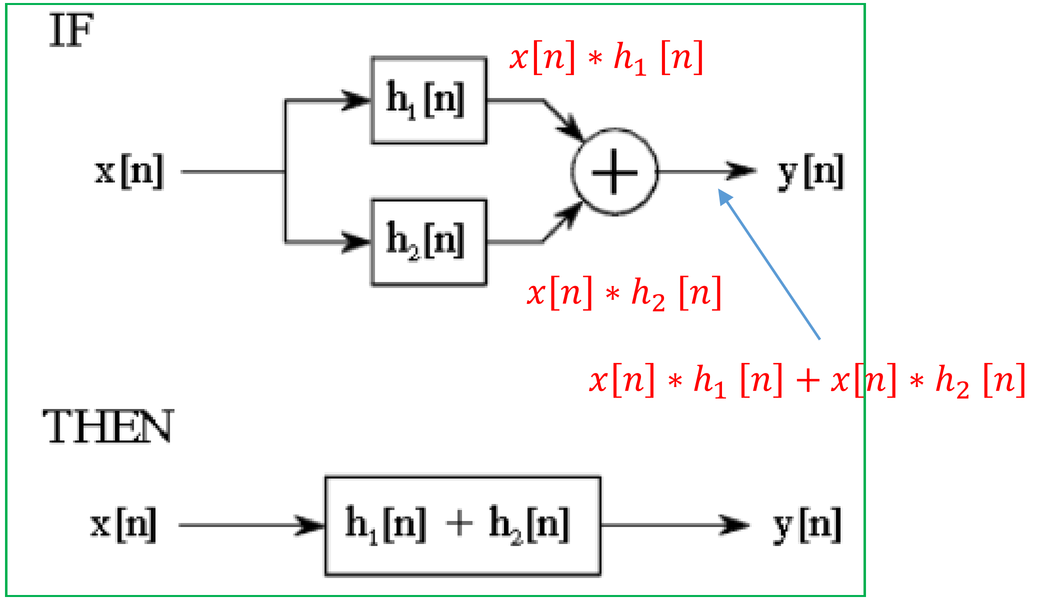
Convolution is distributive over addition,

in discrete time

$$x[n]*(h_1[n]+h_2[n]) = x[n]*h_1[n]+x[n]*h_2[n]$$

in continuous time

$$x(t)*[h_1(t)+h_2(t)] = x(t)*h_1(t)+x(t)*h_2(t).$$



Example:

y[n] = x[n] * h[n] $\qquad x[n]=\left(\frac{1}{2}\right)^n u[n]+2^n u[-n]$ and $h[n]=u[n]$

x[n] in nonzero for entire n, so direct convolution is difficult. Therefore, we will use commutative property.

$$y[n]=x[n]*h[n]=(\boxed{x_1[n]+x_2[n]})*h[n]=(x_1[n]*h[n]+x_2[n]*h[n])=y_1[n]+y_2[n]$$

$1 for\ k \geq 0 \qquad 1 for\ k \leq n$

$$\boxed{y_1[n]}=x_1[n]*h[n]=\sum_{k=-\infty}^{\infty} x_1[k]h[n-k]=\sum_{k=-\infty}^{\infty}\left(\frac{1}{2}\right)^k u[k]u[n-k]=\left(\frac{1-(1/2)^{n+1}}{1-(1/2)}\right)u[n]=2\left(1-(1/2)^{n+1}\right)u[n]$$

$l=-k \qquad l=m-n$

$$\sum_{k=-\infty}^{n} 2^k=\sum_{l=-n}^{\infty}\left(\frac{1}{2}\right)^l=\sum_{m=0}^{\infty}\left(\frac{1}{2}\right)^{m-n}=2^n\sum_{m=0}^{\infty}\left(\frac{1}{2}\right)^m=2^{n+1}$$

$$\boxed{y_2[n]}=x_2[n]*h[n]=\sum_{k=-\infty}^{\infty} 2^k u[-k]u[n-k]=2^{n+1}$$

$1 for\ k \leq 0 \qquad 1 for\ k \leq n$

$$y[n]=y_1[n]+y_2[n]=2\left(1-(1/2)^{n+1}\right)u[n]+2^{n+1}$$

3 Associative Property

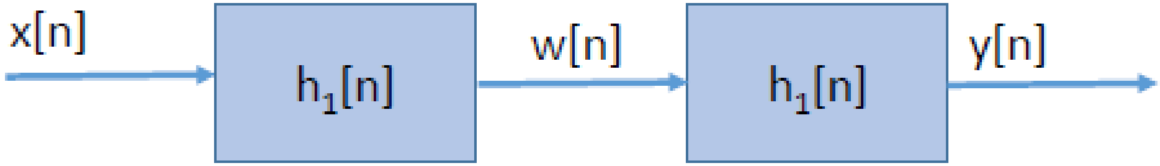
in continuous time

$$x(t)*[h_1(t)*h_2(t)] = [x(t)*h_1(t)]*h_2(t)$$

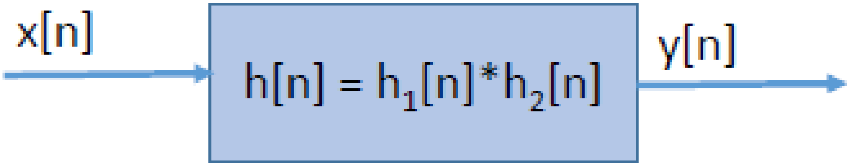
In discrete time

$$x[n]*[h_1[n]*h_2[n]] = [x[n]*h_1[n]]*h_2[n]$$

(A)



(B)



Proof:

From (A), $y[n] = w[n]*h_2[n] = (x[n]*h_1[n])*h_2[n]$

From (B), $y[n] = x[n]*h[n] = x[n]*(h_1[n]*h_2[n])$

4 LTI Systems With and Without Memory

A system is *memory-less* if its output at any time depends only on the value of the input at that same time.

System output: $$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

$y[n]$ depends on only $x[n]$ only if $\mathrm{k} = n$, so for $h[n] = 0$ for $n \neq 0$

A discrete-time LTI system can be memory-less if only: $h[n] = 0, \quad \text{for } n \neq 0$

impulse response $x[n] = \delta[n]$

$y[n] = x[n]\, h[0] = K\delta[n]$

Thus, the impulse response have the form: $h[n] = K\delta[n]$, *with* $K = h[0]$ *is a constant*

the convolution sum reduces to $y[n] = Kx[n]$

If $\mathrm{k} = 1$, then the system is called *identity system*.

Similarly for continuous LTI systems.

a continuous-time LTI system is memory-less if $h(t) = 0 \text{ for } t \neq 0,$

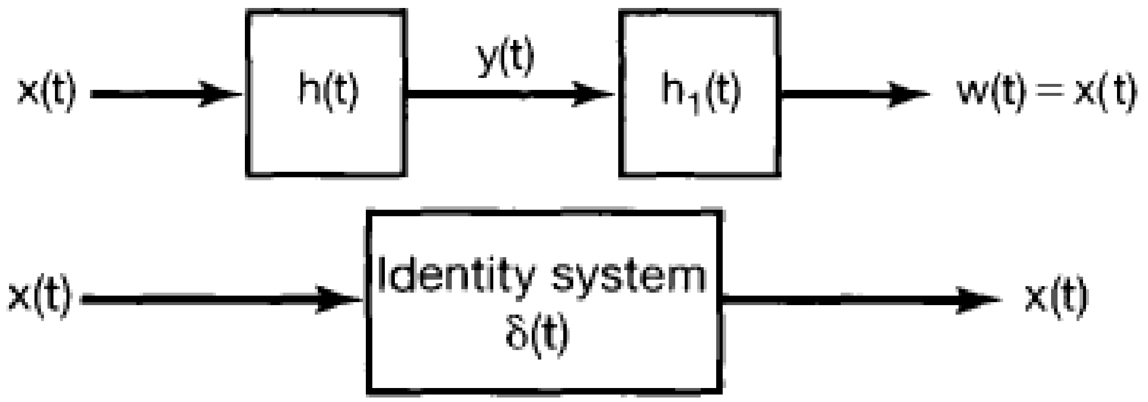
$$h(t) = K\delta(t). \qquad\qquad y(t) = Kx(t)$$

5 Invertibility of LTI Systems

A system is invertible only if *an inverse system exists*

The system with impulse response $h_1(t)$ is inverse of the system with impulse response $h(t)$ if:

$$h(t) * h_1(t) = \delta(t)$$



Example:

Consider the LTI system consisting of a pure time shift $y(t) = x(t - t_0)$

$t_0 > 0$ *delay*

$t_0 < 0$ *advance*

The *impulse response* for the system (for $x(t) = \delta(t)$): $h(t) = \delta(t - t_0)$ ← impulse response $x(t) = \delta(\mathrm{t})$

the system's output (*the convolution*): $y(t) = x(t) * h(t) = x(t) * \delta(t - t_0) = x(t - t_0)$

To recover the input from the output (*invert the system*), all that is required is to shift the output back.

The inverse system impulse response: $h_1(t) = \delta(t + t_0)$

then $h(t) * h_1(t) = \delta(t - t_0) * \delta(t + t_0) = \delta(t)$ ← *identity system* ($y(t) = x(t) * \delta(t) = x(t)$)

Invertibility of LTI Systems: Example 2

Consider an LTI system with impulse response: $h[n] = u[n]$

$u[n-k] = 0 \; for \; k > n$

summer or accumulator

Response of this system (convolution sum): $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, u[n-k]$ $\Rightarrow$ $y[n] = \sum_{k=-\infty}^{n} x[k]$

first difference equation

Inverse system

$\Rightarrow$ $y[n] = x[n] + \sum_{k=-\infty}^{n-1} x[k]$ $\Rightarrow$ $y[n] = x[n] + y[n-1]$ $\Rightarrow$ $x[n] = y[n] - y[n-1]$ $\Rightarrow$ $y[n] = x[n] - x[n-1]$

Impulse response ($x[n] = \delta[n]$): $h_1[n] = \delta[n] - \delta[n-1]$

Verification: $h[n] * h_1[n] = \delta[n]$

$$
\begin{aligned}
h[n] * h_1[n] &= u[n] * \{\delta[n] - \delta[n-1]\} \\
&= \{u[n] * \delta[n]\} - \{u[n] * \delta[n-1]\} \\
&= u[n] - u[n-1] \\
&= \delta[n]
\end{aligned}
$$

$\Rightarrow$ $h[n] * h_1[n] = \delta[n]$

the impulse responses are inverses of each other

6 Causality of LTI Systems

- The output of a *causal system* depends only on the present and past values of the input to the system.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$y[n]$ must not depend on $x[k]$ for $k > n$ to be *causal*

Therefore, for a discrete-time LTI system to be causal: $x[k]h[n-k] = 0$ for $k > n$ $\Rightarrow$ $h[n-k] = 0$ for $k > n$

$for\ k > n \rightarrow n - k < 0$ $\Rightarrow$ $\boxed{h[n] = 0 \quad \text{for } n < 0}$

Causality for LTI system is equivalent to the condition of initial rest (output must be 0 before applying the input)

$for\ k > n\ h[n-k] = 0$ $\qquad$ $for\ k < 0\ h[k] = 0$

$$y[n] = \sum_{k=-\infty}^{n} x[k]h[n-k] = \sum_{k=0}^{\infty} h[k]x[n-k]$$

Both the accumulator ($h[n] = u[n]$) and its inverse ($h[n] = \delta[n] - \delta[n-1]$) are causal.

Inverse system of the accumulator

$$\begin{aligned} h[n] * h_1[n] &= u[n] * \{\delta[n] - \delta[n-1]\} \\ &= u[n] * \delta[n] - u[n] * \delta[n-1] \\ &= u[n] - u[n-1] \\ &= \delta[n]. \end{aligned}$$

- Similarly, for a continuous-time LTI system to be causal:

$$y(t) = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau$$

7 Stability of LTI Systems

- A *system is stable* if every bounded input produces a bounded output (*BIBO*).

Consider, an input x[n] to an LTI system that is bounded in magnitude:

$$|x[n]| < B, \quad \text{for all } n$$

Suppose that we apply this to the LTI system with impulse response h[n].

We take $x[n] = B$

$$|y[n]| = \left|\sum_{k=-\infty}^{\infty} h[k]x[n-k]\right| \leq \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]| \leq B\sum_{k=-\infty}^{\infty} |h[k]| \quad \text{for all } n$$

Therefore, if $\displaystyle\sum_{k=-\infty}^{\infty} |h[k]| < \infty, \;\; \text{then } |y[n]| < \infty$

The system is stable if the *impulse response* $h[n]$ is absolutely summable.

- Similar case in continuous-time LTI system.

the system is stable if the impulse response is absolutely integrable.

Example:

An LTI system with pure time shift is stable.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |\delta[n-n_0]| = 1$$

An accumulator (DT domain) system is unstable.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |u[n]| = \sum_{n=0}^{\infty} |u[n]| = \infty$$

Similarly, an integrator (CT domain) system is unstable.

8 Unit Step Response of An LTI System

- the *unit step response*, $s[n]$ or $s(t)$, the output corresponding to the input $x[n] = u[n]$ or $x(t) = u(t)$.
- it is worthwhile relating the *unit step response* to the impulse response

commutative property

$$s[n] = u[n] * h[n] = h[n] * u[n]$$

Response to the input *h[n]* of a LTI system with unit impulse response *u[n]*.

$u[n]$ is the *unit impulse response* of the *accumulator*.

impulse response of an accumulator

$$h[n] = \sum_{k=-\infty}^{n} \delta[k] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} = u[n]$$

Discrete-time domain

$$\Rightarrow s[n] = \sum_{k=-\infty}^{n} h[k]$$

Running Sum

$$\Rightarrow h[n] = s[n] - s[n-1]$$

First Difference

Continuous-time domain

$$s(t) = \int_{-\infty}^{t} h(\tau) d\tau$$

Running Integral

$$h(t) = \frac{ds(t)}{ds} = s'(t)$$

First Derivative

LTI Systems Described by Differential Equation

(Linear Constant-Coefficient Differential Equation)

A general N[th]-order linear constant-coefficient differential equation that relates the input $x(t)$ to the output $y(t)$ is given by

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

Example1:

consider a first-order differential equation $\frac{dy(t)}{dt} + 2y(t) = x(t)$

where the input to the system is: $x(t) = Ke^{3t}u(t)$

Solve for y(t).

The complete solution is $y(t) = y_p(t) + y_h(t)$

$y_p(t)$ *the particular solution*

$y_h(t)$ *The homogeneous solution,*

- Finding the particular solution $y_p(t)$ (signal of the same form as the input)

forced response $y_p(t) = Ye^{3t}$ ← Determine Y

From differential equation:

$$3Ye^{3t} + 2Ye^{3t} = Ke^{3t} \Rightarrow 3Y + 2Y = K \Rightarrow Y = \frac{K}{5}$$

$$y_p(t) = \frac{K}{5}e^{3t}, K\ real\ and\ t > 0$$

- Finding the homogeneous solution(hypothesize a solution)

$y_h(t) = Ae^{st}$ ← Determine s and A

From differential equation:

$$sAe^{st} + 2Ae^{st} = 0 \Rightarrow A(s+2)e^{st} = 0 \Rightarrow s = -2$$

$$y_h(t) = Ae^{-2t}$$

Complete solution:

$$y(t) = \frac{K}{5}e^{3t} + Ae^{-2t}$$

Example_contd

- To find A suppose that the auxiliary condition is $y(0) = 0$, i.e. , at $\mathrm{t} = 0, y(t) = 0$

Using this condition into the complete solution, we get:

$$y(t) = \frac{K}{5}e^{3t} + Ae^{-2t} \quad \text{With} \quad y(0) = 0$$

$$\Rightarrow \quad 0 = \frac{K}{5} + A \Rightarrow A = -\frac{K}{5}$$

$$\Rightarrow \quad y(t) = \frac{K}{5}\left[e^{3t} - e^{-2t}\right], \quad t > 0$$

$$= \frac{K}{5}\left[e^{3t} - e^{-2t}\right]u(t)$$

Example2:

Find $y[n]$of the system with the difference equation $\quad y[n] - \frac{1}{2}y[n-1] = x[n]$

We have the output $y[n] = x[n] + \frac{1}{2}y[n-1]$

Consider the input $x[n] = k\,\delta[n]$ and initial condition $y[-1] = 0$ (rest)

$$y[0] = x[0] + \frac{1}{2}y[-1] = k$$

$$y[1] = x[1] + \frac{1}{2}y[0] = \frac{1}{2}k$$

$$y[2] = x[2] + \frac{1}{2}y[1] = \left(\frac{1}{2}\right)^2 k$$

$$\vdots$$

$$y[n] = x[n] + \frac{1}{2}y[n-1] = \left(\frac{1}{2}\right)^n k$$

Setting $k = 1$ we obtain the *impulse response* for the system

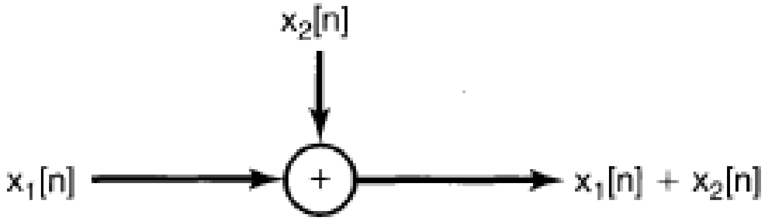
$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

impulse response with infinite duration

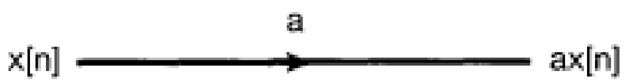
$\Rightarrow$ *infinite impulse response (IIR) systems*.

Block Diagram Representations of Systems

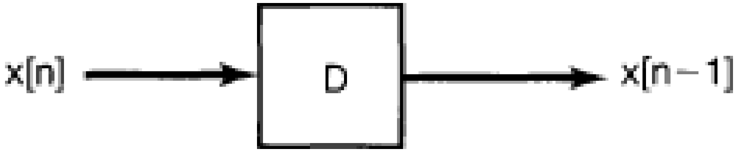
systems described by linear constant -coefficient difference and differential equations can be represented in terms of block diagram interconnections of elementary operations (adder, scaler, unit delay).



adder



scaler

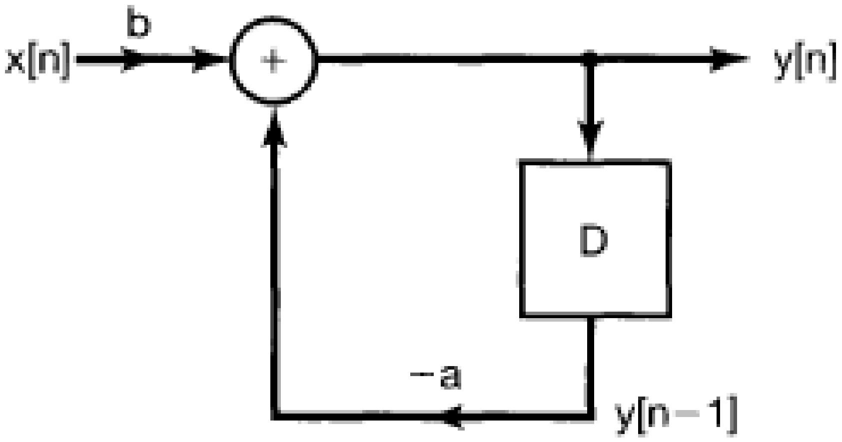


unit delay

Example:

Consider the causal system described by the first-order difference equation

$$y[n] + a\, y[n-1] = b\, x[n]$$



Consider the causal continuous−time system described by a first−order differential equation

$$\frac{dy(t)}{dt} + ay(t) = b\, x(t)$$