

Midterm2-280

Question1(2+3+3)

- a) Show that the equation $x2^{x^2+2x} = 6$ has a solution in $(0,1)$.
- b) Show that $f(x) = \frac{1}{x^2+4}$ is uniformly continuous on $\mathbb{R}$
- c) Show that $f(x) = x + \frac{4}{x}$ and $x > 0$ has a minimum at $x = 2$

Question2 (3+3+3)

- a) Show that $\frac{1}{x} > \cot x$ holds on $(0, \frac{\pi}{2})$
- b) Show that $(\sin^{-1})'(x) = \frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$
- c) Find $\lim_{x \rightarrow 0} \frac{1 - \cos x - \frac{x^2}{2} + \frac{x^4}{24}}{\sin x - x + \frac{x^3}{6}}$

Question3 (3+2+3)

- a) Show that for $x > 0$
- $$|\ln(1+x) - (x - \frac{x^2}{2} + \frac{x^3}{3})| \leq \frac{x^4}{4}$$
- b) Approximate $\cos x$ on $[-1,1]$ by a polynomial of degree 4
And estimate the error.
- c) Let $f [0,2] \rightarrow \mathbb{R}$ be continuous and such that $f(0) = f(2)$
Show that there exists $c \in [0,1]$ such that $f(c) = f(c+1)$