

**Question 1(6).** Determine the sup, max, inf and min of the following sets:

$$A = \left\{ \frac{1}{\frac{x^2}{3} + 3^x}, \quad x > 0, \quad x \in \mathbb{R} \right\}$$

$$B = \left\{ \frac{2n}{n+1} + \frac{1}{2n} + \frac{1}{2}, \quad n \in \mathbb{N} \right\}$$

**Question2 (6).**

a) Decide whether the sequence  $x_n = \left(\frac{e}{3}\right)^n + \frac{3n+1}{2n+1}$  is bounded.

b) Which of the following two sequences is Cauchy:  $a_n = \frac{n^2+1}{n^2}$  and  $b_n = n^2 + \frac{1}{n^2}$

c) Using the Definition show that  $\lim_{n \rightarrow \infty} \frac{4n}{5n+2} = \frac{4}{5}$

**Question3 (2+2+2+3).** Find  $\lim_{n \rightarrow \infty} x_n$  if,

a)  $x_n = (-1)^n \frac{\sqrt{n+\pi} \sin \sqrt{n+\pi}}{n}$

b)  $x_n = (-1)^{n+1} \left(1 + \frac{1}{n}\right)$

c)  $x_n = \frac{2^n + e^n + 3^n}{3^n}$

d)  $x_n = \frac{n!}{n^n}$

**Question4 (2+2).** Determine whether the series converges. If so find its sum:

a)  $\sum_{n=1}^{\infty} \frac{n^n}{2^n n!}$       b)  $\sum_{n=0}^{\infty} \frac{e^{n+1}}{\pi^{n-1}}$