King Saud University: Mathematics Department Math-254
Summer Semester 1430-31 H Midterm Examination
Maximum Marks = 35 TIMe: 120 Mins.

Name of the Student:———————— I.D. No. ————————

Name of the Teacher:———————— Section No. ————————

The Answer Table for Q.1 to Q.10 : Marks: 2 for each one ($2 \times 10 = 20$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Question No.	Marks
Q. 1 to Q. 10	
Q. 11	
Q. 12	
Q. 13	
Total Marks	

Question 1: The second approximation to the root of the equation $x^3 = x^2 + 1$ in $[1, 2]$ by the Bisection method is:

(a) $c_2 = 1.75$ (b) $c_2 = 1.5$ (c) $c_2 = 1.25$ (d) $c_2 = 1.625$

Question 2: The equivalent form $f(x) = 0$ of the nonlinear equation $g(x) = \dfrac{2x^3 - 2}{3x^2 - 2}$ is:

(a) $x^3 + 3x + 2 = 0$ (b) $x^3 - 2x + 2 = 0$ (c) $x^3 - 3x + 2 = 0$ (d) $x^3 - 2x - 2 = 0$

Question 3: Newton's iterative formula for approximation to the square root of a real number R is:

(a) $x_{n+1} = \dfrac{x_n}{2R}$ (b) $x_{n+1} = \dfrac{1}{2}(3x_n - \dfrac{R}{x_n})$ (c) $x_{n+1} = \dfrac{3Rx_n}{2}$ (d) $x_{n+1} = \dfrac{1}{2}(x_n + \dfrac{R}{x_n})$

Question 4: The first approximation of the root of $x^2 = 4$ using Newton's iterative formula, if $x_0 = 3$; is:

(A) 2.067 (b) 1.5 (c) 3.5 (d) 2.167

Question 5: The first approximation of the root of $x^3 + 4x^2 = 10$ using modified Newton's iterative formula, if $x_0 = 1.5$; is:

(a) $x_1 = 1.46$ (b) $x_1 = 1.36$ (c) $x_1 = 1.56$ (d) $x_1 = 1.66$

Question 6: If the iterative scheme $x_{n+1} = ax_n^2 + \dfrac{2b}{x_n} - 8$, $n \geq 0$ converges quadratically to 1, then the values of a and b are:

(a) -2 and 2 (b) 1 and 1 (c) 3 and 3 (d) -3 and -2

Question 7: The order of convergence of Newton's method to the root $\alpha = 3$ of the equation $(x-3)^3 e^{(x-3)} = 0$ is:

(a) of order 2 (b) of order 4 (c) of order 3 (d) of order 1

Question 8: If the linear system $\begin{array}{rcl} 6x - 4y & = & 2 \\ -3x + 2y & = & k \end{array}$ has infinitely many solutions, then the value of k is:

(a) $k = 4$ (b) $k = -1$ (c) $k = -4$ (d) $k = 1$

Question 9: Determinant of the matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 3 \\ 2 & 2 & 2 \end{pmatrix}$ by LU decomposition ($l_{ii} = 1$) is:

(a) 18 (b) 28 (c) 8 (d) -8

Question 10: The value of α for which the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & \alpha \end{pmatrix}$ is singular, is:

(a) $\alpha = 1$ (b) $\alpha = 2$ (c) $\alpha = 3$ (d) $\alpha = 1.5$

Question 11: Develop an iterative formula for the root of any positive number N using Scant method. Then use it to find the first approximation of the fifth root of 32 using $x_0 = 1.0$ and $x_1 = 1.5$. Find the absolute error. [5 points]

Question 12: Use Newton's method to find the first approximation to the roots of the following nonlinear system with $x_0 = 1$ and $y_0 = -1$: [5 points]

$$\begin{aligned} x^3 + 3y^2 &= 2 \\ x^2 + 2y &= -2 \end{aligned}$$

Question 13: Use LU decomposition method with **Crout's method** ($u_{ii} = 1$) to find the unique solution to the following linear system: [5 points]

$$\begin{aligned} x + 2y + 3z &= 1 \\ 6x + 5y + 4z &= -1 \\ 2x + 5y + 6z &= 5 \end{aligned}$$