King Saud University: Mathematics Department M-254
Semester II (2nd Midterm Exam) 1439-1440 H
Maximum Marks=25 Time Allowed: 90 Mins.

Questions : $(5 + 5 + 5 + 5 + 5)$

Q1: Use LU-factorization with Doolittle's method ($l_{ii} = 1$) to find the value of α for which the following linear system has infinitely many solutions, and write down this solution.

$$\begin{aligned} x_1 + x_2 &= 5/9 \\ 3x_1 + \alpha x_2 + 5x_3 &= 0 \\ 7x_2 + 3x_3 &= -1 \end{aligned}$$

Q2: Rearrange the following linear system of equations

$$\begin{aligned} x_1 + 6x_2 - 3x_3 &= 4 \\ 2x_1 + 2x_2 + 6x_3 &= 7 \\ 5x_1 + 2x_2 - x_3 &= 6 \end{aligned}$$

such that the convergence of Jacobi iterative method is guaranteed. Then, use the initial solution $\mathbf{x}^{(0)} = [0, 0, 0]^T$, compute the second approximation $\mathbf{x}^{(2)}$. Also, compute an error bound for the error $\|\mathbf{x} - \mathbf{x}^{(10)}\|$.

Q3: If $\mathbf{x}^* = [-1.99, 2.99]^T$ is an approximate solution of the linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1/2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix},$$

then find an upper bound for the relative error.

Q4: Let $f(x) = \sqrt{x - x^2}$ and $p_2(x)$ be the quadratic Lagrange interpolating polynomial which interpolates f at $x_0 = 0, x_1 = \alpha$ and $x_2 = 1$. Find the largest value of α, in the interval $(0, 1)$, for which

$$f(0.5) - p_2(0.5) = -0.25.$$

Q5: Let $f(x) = (x + 1)\ln(x + 1)$ be the function defined over the interval $[1, 2]$. Compute the error bound for fifth degree Lagrange interpolating polynomial for equally spaced data points for the approximation of $(2.9 \ln 2.9)$.