

Questions:

(6 + 6 + 7 + 6) Marks

Question 1:

Use Gauss elimination by partial pivoting to find the inverse A^{-1} involving $\alpha (\neq 2)$ of the following matrix,

$$A = \begin{pmatrix} 1 & \alpha & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}.$$

Then use A^{-1} with $\alpha = 3$ to solve the given linear system $Ax = [2, 2, 3]^T$.

Question 2:

Use LU decomposition by Doolittle's method to find the value(s) of $\alpha (\neq 2)$ for which the following matrix

$$A = \begin{pmatrix} 2 & \alpha & -1 \\ \alpha & 2 & 1 \\ -1 & 1 & 1 \end{pmatrix},$$

is singular. Compute the unique solution of the linear system $Ax = [1, 0, -1/2]^T$ by using the largest negative integer value of α .

Question 3:

Consider the following linear system

$$\begin{aligned} x_1 + 2x_2 &= 3 \\ x_1 + 3x_2 + x_3 &= 5 \\ -2x_2 + 3x_3 &= 1 \end{aligned}$$

Find the Gauss-Seidel iteration matrix T_G and show that $\|T_G\|_\infty > 1$. Use Gauss-Seidel method to compute the approximate solution of the system within accuracy 5×10^{-2} , using the initial solution $x^{(0)} = [0.9, 0.9, 0.9]^T$. If $x = [1, 1, 1]^T$ is the exact solution of the linear system, then compute the absolute error.

Question 4:

Consider a nonsingular linear system $Ax = b$, where

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 4/3 & 1 & -8/3 \\ -1/3 & 0 & 2/3 \\ -2/3 & -1 & 7/3 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

If b of the given linear system is changed to $b^* = [1, 1, 1.99]^T$, then how large a relative error can this change produce in the solution to the given linear system $Ax = b$?