King Saud University: Mathematics Department Math-254
First Semester 1446 H Second Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Questions: (6 + 7 + 6 + 6) Marks

Question 1:

Compute the determinant of the matrix A and the solution of the following linear system using the Gaussian elimination with partial pivoting

$$\begin{aligned} x_1 + x_2 + x_3 &= 0.5 \\ 2x_1 - 3x_2 + x_3 &= -1 \\ -x_1 - 1.5x_2 + 2.5x_3 &= -1 \end{aligned}$$

Question 2:

Use LU-factorization method with Doolittle's method ($l_{ii} = 1$) to find the solution of the consistent system for $\alpha \neq 3$.

$$\begin{aligned} x_1 + x_2 &= 1 \\ 3x_1 + \alpha x_2 + 5x_3 &= 8 \\ 7x_2 + 3x_3 &= 3 \end{aligned}$$

Question 3:

Consider the following linear system of equations

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ x_1 + 8x_2 + x_3 &= 10 \\ x_2 + 2x_3 &= 3 \end{aligned}$$

Find the matrix form of Gauss-Seidel method. If $\mathbf{x^{(0)}} = [0.5, 0.5, 0.5]^T$, then compute an error bound $\|x - x^{(10)}\|$.

Question 4:

Let $f(x) = e^{3x} + \cos 2x$ (x is in radian) and $x_0 = 0.1$, $x_1 = 0.2$, $x_2 = 0.4$, $x_3 = 0.5$. Then find the best approximation of $f(0.45)$ using the quadratic Lagrange interpolation formula and also estimate an error bound and the absolute error for the approximation.