and the l_∞-norm of residual vector, $\|\mathbf{r}\|_\infty = 0.01$. From the equation relative error, we get

$$\frac{\|\delta\mathbf{x}\|}{\|\mathbf{x}\|} \leq \frac{25(0.01)}{2} = 0.1250,$$

the possible relative error in the solution to the given linear system. •

Question 4: Use the following table to find the best approximation of $f(0.6)$ by using quadratic Lagrange interpolating polynomial for equally spaced data points [6 Marks]

x	0.15	0.2	0.3	0.5	0.55	0.8	1
$f(x)$	-0.0427	-0.0644	-0.1084	-0.1733	-0.1808	-0.1428	0

The above table is for $f(x) = x^2 \ln x$. Determine the number of points when the error for quadratic Lagrange interpolation for equally spaced data points is to be bounded by 10^{-6}.

Solution. Since given $x = 0.6$, therefore, the best points for the equally spaced data points Lagrange quadratic polynomial are, $x_0 = 0.3, x_1 = 0.55$ and $x_2 = 0.8$ with $h = 0.25$. Consider the quadratic Lagrange interpolating polynomial as

$$f(x) = p_2(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2), \tag{1}$$

$$f(0.6) \approx p_2(0.6) = L_0(0.6)(-0.1084) + L_1(0.6)(-0.1808) + L_2(0.6)(-0.1428). \tag{2}$$

The Lagrange coefficients can be calculate as follows:

$$\begin{aligned} L_0(0.6) &= \frac{(0.6-0.55)(0.6-0.8)}{(0.3-0.55)(0.3-0.8)} = -2/25 = -0.08, \\ L_1(0.6) &= \frac{(0.6-0.3)(0.6-0.8)}{(0.55-0.3)(0.55-0.8)} = 24/25 = 0.96, \\ L_2(0.6) &= \frac{(0.6-0.3)(0.6-0.55)}{(0.8-0.3)(0.8-0.55)} = 3/25 = 0.12. \end{aligned}$$

Putting these values of the Lagrange coefficients in (2), we have

$$f(0.4) \approx p_2(0.4) = (-2/25)(-0.1084) + (24/25)(-0.1808) + (3/25)(-0.1428) = -0.1820,$$

which is the required approximation of the given exact solution $0.36 \ln 0.6 \approx -0.1839$.

To compute number of points we have to use error bound for the quadratic Lagrange interpolation for equally spaced data points as

$$|f(x) - p_2(x)| \leq \frac{Mh^3}{9\sqrt{3}} \leq 10^{-6}.$$

As $h = \dfrac{b-a}{n} = \dfrac{0.8-0.3}{n} = \dfrac{0.5}{n}$, so

$$\frac{M(0.5)^3}{(9\sqrt{3})n^3} \leq 10^{-6}.$$

Since

$$M = \max_{0.3 \le x \le 0.8} |f^{(3)}(x)|,$$

and the first three derivatives are

$$f'(x) = 2x \ln x + x, \qquad f''(x) = 2 \ln x + 3, \qquad f^{(3)}(x) = \frac{2}{x},$$

so

$$M = \max_{0.3 \le x \le 0.8} \left| \frac{2}{x} \right| = 20/3 = 6.6667.$$

Hence

$$n^3 \ge \frac{(6.6667)(0.5)^3(10^6)}{9\sqrt{3}},$$

or

$$n \ge \left(\frac{(6.6667)(0.5)^3(10^6)}{9\sqrt{3}} \right)^{1/3},$$

which gives

$$n \ge 37.6709, \qquad \text{gives} \qquad n = 38,$$

and so the number of points is, $n + 1 = 38 + 1 = 39$. •