King Saud University: Math. Dept. M-254
Semester II (1st Midterm Makeup Exam) 1439-1440H
Max Marks=25 Time Allowed: 90 Mins.

Questions: (5+5+5+5+5)

(1) Which of the following iterations

$$(i) \quad x_{n+1} = e^{x_n} - x_n - 1, \quad n \geq 0 \quad (ii) \quad x_{n+1} = \ln(2x_n + 1), \quad n \geq 0$$

is most suitable to approximate the root of the equation $e^x - 2x = 1$ in the interval $[1, 2]$? Starting with $x_0 = 1.5$, find the second approximation x_2 of the root. Also, compute the error bound for the approximation.

(2) Successive approximations x_n to the desired root are generated by the scheme

$$x_{n+1} = e^{x_n} - 2, \qquad n \geq 0$$

Find $f(x_n)$ and its derivative $f'(x_n)$ and then use Newton's method to find the first approximation of the root, starting with $x_0 = 10$.

(3) Derive an iterative scheme to find the cubic root of a number based on *Newton's method*. Find the first approximation to the cubic root of $a = 155$, with $p_0 = 5$.

(4) Show that the iterative procedure for evaluating the reciprocal of a positive number N using the secant method is

$$x_{n+1} = x_n + (1 - N)x_{n-1}, \quad n = 1, 2, \cdots$$

Then use this formula to find the first approximation to the reciprocal of 5 using $x_0 = 0.05$ and $x_1 = 0.1$.

(5) Find the first approximation for the nonlinear system

$$\begin{aligned} x^3 + 3y^2 &= 21 \\ x^2 + 2y &= -2 \end{aligned}$$

using Newton's method, starting with initial approximation $(x_0, y_0)^T = (1, -1)^T$.