King Saud University: Mathematics Department Math-254
Summer Semester 1439-40 H First Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Question 1: Develop the iterative formula

$$x_{n+1} = \frac{x_n^2 - b}{2x_n - a}, \qquad n \geq 0,$$

for the approximate roots of the quadratic equation $x^2 - ax + b = 0$ using the Newton's method. Then use the develop formula to find the second approximation of the positive root of $x^2 - 3x = 4$, starting with $x_0 = 3.5$. [5 Marks]

Question 2: Successive approximations x_n to the desired root are generated by the scheme

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 3x_n^2}{e^{x_n} + 6x_n}, \qquad n \geq 0.$$

Find function $f(x_n)$ and then use Secant method to find the second approximation of the root starting with $x_0 = -0.5$ and $x_1 = -0.25$. [5 Marks]

Question 3: The equation $1 - 2\cos(x) + \cos^2(x) = 0$ has the root $\alpha = 0$. Develop the Modified Newton's formula for computing this root, then use it to find the second approximation starting with an initial approximation $x_0 = 0.5$. Show that the developed formula converges at least quadratically. [5 Marks]

Question 4: Use the simple Gaussian elimination method, find all values of a and b for which the following linear system is consistent or inconsistent. Find the solutions when the system is consistent. [5 Marks]

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 4 \\ 2x_1 - 3x_2 + ax_3 &= 5 \\ 3x_1 - 4x_2 + 5x_3 &= b \end{aligned}$$

Question 5: Find determinant of the coefficient matrix A of the following linear system using LU decomposition by Crout's method. If $\det(A) = |A| = -1$, then find the unique solution of the following system. [5 Marks]

$$\begin{aligned} x_1 - x_3 &= 1 \\ x_2 + x_3 &= 1 \\ -x_1 + x_2 + \alpha x_3 &= 1 \end{aligned}$$