King Saud University: Mathematics Department Math-254
First Semester 1447 H First Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Questions: (6 + 6 + 6 + 7) Marks

Question 1:

Show that Newton's iterative scheme to find the approximation to the root $\alpha = 0$ of the following nonlinear equation

$$xe^{-x} = 0,$$

is

$$x_{n+1} = x_n - \frac{x_n}{(1 - x_n)}, \qquad n \geq 0.$$

Use this iterative scheme to find the second approximation when $x_0 = 0.2$. Show that rate of convergence for the given scheme is quadratic.

Question 2:

Use secant method to find the second approximation, using $x_0 = 1$ and $x_1 = 2$, of the value of x that produces the point on the graph of $y = \frac{1}{x}$ that is closest to the point $(2, 1)$. Also, find the approximation of the point.

Question 3:

Use quadratic convergence method to find the second approximation of the root $\alpha = 1$ of the following nonlinear equation

$$(x - 1)^2 \sin x = 0, \quad (x \textit{ in radian}),$$

using $x_0 = 1.5$. Compute the relative error.

Question 4:

(a) Use the simple Gaussian elimination method to find the inverse A^{-1} in α $(\neq -1)$ of the following matrix

$$A = \begin{pmatrix} 1 & -1 & 2\alpha \\ 2 & 2 & 4 \\ 0 & 4 & 8 \end{pmatrix}.$$

(b) Find the unique solution of the system $Ax = [-4, 8, 12]^T$ by taking the largest possible negative integer value of α in A^{-1} got in part (a).