King Saud University: Mathematics Department Math-254
First Semester 1446 H First Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Questions: (6 + 6 + 7 + 6) Marks

Question 1:

Show that the secant method for finding approximation of the square root of a positive number N is

$$x_{n+1} = \frac{x_n x_{n-1} + N}{x_n + x_{n-1}}, \quad n \geq 1.$$

Carry out the first three approximations for the square root of 9, using $x_0 = 2, x_1 = 2.5$ and also compute the absolute error.

Question 2:

Find a nonlinear equation and show that it has a root $(25)^{1/3}$ in the interval $[2.0, 3.0]$. Use $x_0 = 2$ and the quadratic convergent numerical method to compute the second approximation to the root. Find the relative error.

Question 3:

The modified Newton's method for the approximation of the root $\alpha = 0$ of the nonlinear equation $(1 - \cos(x))^2 = 0$ is,

$$x_{n+1} = x_n - 2\frac{\sin(x_n)}{(1 + \cos(x_n))}, \quad n \geq 0.$$

Use this iterative form to find the absolute error $|\alpha - x_2|$ using $x_0 = 0.5$. Show that the given iterative method converges atleast quadratically to the root $\alpha = 0$.

Question 4:

Suppose that $f'''(x)$ exists, with $f(\alpha) = 0$, $f'(\alpha) \neq 0$, and $f''(\alpha) \neq 0$. Show that the rate of convergence of the Newton's method is only quadratic.