King Saud University: Mathematics Department M-254
Semester I (1st Midterm Exam) 1440-1441 H
Maximum Marks=25 Time Allowed: 90 Mins.

Questions : (5 + 5 + 5 + 5 + 5)

Q1: Use Newton's method with $x_0 = 0$ to find the second approximation of the value of x that produces the point on the graph of $y = x^2$ that is closest to the point $(3, 2)$. What is the value of the point (x, y) on the graph of $y = x^2$.

Q2: Show that the rate of convergence of the Newton's method at the root $x = 0$ of the equation $x^2e^x = 0$ is linear. Use quadratic convergence method to find second approximation to the root using $x_0 = 0.1$. Also, compute the absolute error.

Q3: Show that the secant method for finding approximation of the cubic root of a positive number N is

$$x_{n+1} = \frac{x_n x_{n-1}(x_n + x_{n-1}) + N}{x_n^2 + x_n x_{n-1} + x_{n-1}^2}, \quad n \geq 1.$$

Carry out the first two approximations for the cubic root of 27, using $x_0 = 2, x_1 = 2.5$ and also compute absolute error.

Q4: Find the first approximation for the nonlinear system

$$\begin{aligned} x^2 + y^2 &= 1 \\ \tfrac{1}{3}x^2 + \tfrac{1}{2}y^2 &= 1 \end{aligned}$$

using the Newton's method, starting with initial approximation $(x_0, y_0)^T = (1, 1)^T$.

Q5: The equation $2^x - 5x + 1 = 0$ can be converted to a fixed-point problem as

$$x = \frac{1}{1+c}\left(cx + \frac{2^x + 1}{5}\right),$$

with c a constant. Find a value of c to ensure rapid convergence of the following scheme near $x = 0.1$

$$x_{n+1} = \frac{1}{1+c}\left(cx_n + \frac{2^{x_n} + 1}{5}\right), \qquad n \geq 0.$$

Compute the third iterates, starting with $x_0 = 0.1$.