King Saud University: Mathematics Department MAth-254
Second Semester 1447-48 H Final Examination Solution
Maximum Marks = 40 Time: 180 mins.

The Answer Tables for Q.1 to Q.13 : Marks: 2 for each one ($2 \times 13 = 26$)

Ps. : Mark {A,B,C,or D } for the correct answer in the box.(Math)

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13
A,B,C,D	B	A	C	C	A	B	B	A	C	C	A	C	A

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Question 14: Let $f(x) = e^{-x}$ and $x_0 = 0, x_1 = 0, x_2 = 1, x_3 = 1$, find the approximation of $1/e^{0.5}$ by using the quadratic Newton's polynomial $p_2(x)$ and then use approximation by $p_2(x)$ to find the approximation of $1/e^{0.5}$ by using the cubic Newton's polynomial $p_3(x)$. Compute absolute error and the error bound for the approximation by cubic Newton's polynomial.

Solution. Using $f(x) = e^{-x}$ and $x_0 = 0, x_1 = 0, x_2 = 1$, and $x = 0.5$, the best Newton's polynomial is the quadratic Newton's interpolating polynomial which has the following form

$$p_2(x) = f[x_0] + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2].$$

which can be written as using $x = 0.5$

$$p_2(0.5) = f[0] + (0.5 - 0)f[0, 0] + (0.5 - 0)(0.5 - 0)f[0, 0, 1].$$

Now we find the zeroth-order, first-order and second-order divided differences as follows:

$$f[0] = e^{-0} = 1,$$

$$f[0, 0] = \frac{f'(0)}{1!} = -e^{-0} = -1,$$

$$f[0, 0, 1] = \frac{f[0, 1] - f[0, 0]}{1 - 0} = f(1) - f(0) - f'(0) = e^{-1} - e^{-0} - (-e^{-0}) = e^{-1} = 0.3679.$$

Thus

$$1/e^{0.5} \approx p_2(0.5) = 1 + (0.5)(-1) + (0.25)(0.3679) = 0.5920,$$

the required approximation of $1/e^{0.5}$ by $p_(0.5)$. Now we use $p_2(0.5)$ in $p_3(0.5)$ as

$$p_3(0.5) = p_2(0.5) + (0.5 - 0)(0.5 - 0)(0.5 - 1)f[0, 0, 1, 1],$$

where

$$f[0, 0, 1, 1] = \frac{f[0, 1, 1] - f[0, 0, 1]}{1 - 0} = f[0, 1, 1] - f[0, 0, 1] = \frac{f[1, 1] - f[0, 1]}{1 - 0} - e^{-1},$$

or

$$f[0, 0, 1, 1] = f'(1) - (f(1) - f(0)) - e^{-1} = 1 - 3e^{-1} = -0.1036.$$

Thus

$$p_3(0.5) = 0.5920 + (0.5 - 0)(0.5 - 0)(0.5 - 1)(-0.1036) = 0.6049,$$

the required approximation of $1/e^{0.5}$ by cubic Newton's polynomial.
The possible absolute error in the approximation of $p_3(x)$ is

$$|f(0.5) - p_3(0.5)| = |1/e^{0.5} - p_3(0.5)| = |0.6065 - 0.6049| = 0.0016.$$

For the polynomial $p_3(x)$, we have the error formula

$$|f(0.5) - p_2(0.5)| = \frac{|f^{(4)}(\eta(x))|}{4!}|(0.5 - 0)(0.5 - 0)(0.5 - 1)(0.5 - 1)|.$$

The fourth derivative of the given function is given as

$$f^{(4)}(x) = e^{-x} \qquad \text{and} \qquad |f^{(4)}(\eta(x))| = \left|e^{-\eta(x)}\right|, \quad \text{for} \quad \eta(x) \in (0, 1).$$

Then

$$|f^{(4)}(\eta(x))| \leq M = \max_{0 \leq x \leq 1} |e^{-x}| = 1,$$

and so

$$|f(0.5) - p_3(0.5)| \leq ((1)(0.0625))/24 = 0.0026,$$

which is the required error bound for the approximation of $p_3(0.5)$. •

Question 15: Determine the number of subintervals n required to approximate the integral $\int_0^2 \frac{1}{x+4}dx$, with an error less than 10^{-4} using composite Simpson's rule. Then use the value of n to find the absolute errors in the approximations of the given integral by using Trapezoidal and Simpson's rules.

Solution. We have to use the error bound formula of composite Simpson's rule which is

$$|E_{S_n}(f)| \leq \frac{(b-a)}{180}h^4 M \leq 10^{-4}.$$

Given the integrand is $f(x) = \frac{1}{x+4}$, and we have $f^{(4)}(x) = \frac{24}{(x+4)^5}$. The maximum value of $|f^{(4)}(x)|$ on the interval $[0, 2]$ is $\frac{3}{128}$, and thus $M = \frac{3}{128}$. Using the above error formula, we get

$$\frac{3}{(90 \times 128)}h^4 \leq 10^{-4}, \quad \text{or} \quad \frac{3}{(90 \times 128)}((2-0)/n)^4 \leq 10^{-4},$$

(since $h = \frac{(2-0)}{n}$). Thus solving for n

$$n \geq \left(\frac{(16 \times 3 \times 10^4)}{(90 \times 128)}\right)^{1/4} \geq 2.5407,$$

so the number of even subintervals n required is $n = 4$. Thus the approximation of the given integral using $h = \frac{2-0}{4} = \frac{1}{2} = 0.5$ by Tapezoidal rule is

$$I(f) = \int_0^2 \frac{1}{x+4} \approx T_4(f) = \frac{0.5}{2}\Big[f(0) + 2[f(0.5) + f(1) + f(1.5)] + f(2)\Big],$$

$$I(f) = \int_0^2 \frac{1}{x+4} \approx T_4(f) = \frac{1}{4}\Big[0.2500 + 2[0.2222 + 0.2000 + 0.1818] + 0.1667\Big] = 0.4062.$$

and by Simpson's rule is

$$I(f) = \int_0^2 \frac{1}{x+4} \approx S_4(f) = \frac{0.5}{3}\Big[f(0) + 4[f(0.5) + f(1.5)] + 2f(1) + f(2)\Big],$$

$$I(f) = \int_0^2 \frac{1}{x+4} \approx S_4(f) = \frac{1}{6}\Big[0.25 + 4(0.2222 + 0.1818) + 2(0.2) + 0.1667\Big] = 0.4055.$$

Since the exact value of the given integral is

$$Exact\ sol = I(f) = \int_0^2 \frac{1}{x+4}dx = \ln(1.5) = 0.4055,$$

and so the absolute error by Trapezoidal rule is

$$|E_T| = |I(f) - T_4(f)| = |0.4055 - 0.4062| = 0.0007,$$

and by Simpson's rule is

$$|E_S| = |I(f) - S_4(f)| = |0.4055 - 0.4055| = 0.0000,$$

which is equal to the true value of the given integral up to 4 decimal places. •

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(**13 Multiple choice questions and Two (2) Full questions**)

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Quest. No.	Marks Obtained	Marks for Question
Q. 1 to Q. 13		26
Q. 14		7
Q. 15		7
Total		40

Question 1: The first approximation of the square root of 19 using a quadratic convergent method with $x_0 = 5$ is:

(A) 4.44 **(B)** 4.40 **(C)** 4.36 **(D)** None of these

Question 2: The iterative scheme $x_{n+1} = 2 + (3+k)x_n - kx_n^3,\ n \geq 0$ converges at least quadratically to the root $\alpha = -1$ if k is:

(A) $\frac{3}{2}$ (B) 1 (C) $\frac{1}{2}$ (D) None of these

Question 3: Let $A = \begin{bmatrix} 1.001 & 1.5 \\ 2 & 3 \end{bmatrix}$, then the determinant of the lower-triangular matrix L of the LU factorization using Crout's method is:

(A) 2.002 (B) 1.001 (C) 0.003 (D) None of these

Question 4: The l_∞-norm of the first approximation of the solution vector obtained by Gauss-Seidel method starting with $\mathbf{x^{(0)}} = [0, 0, 0]^T$ for the following linear system $4x_1 - x_2 + x_3 = 4, \quad 3x_1 + 8x_2 + x_3 = -21, \quad -2x_1 + x_2 + 5x_3 = 15$; is:

(A) 8 **(B)** 3 **(C)** 4 **(D)** None of These

Question 5: Let $A = \begin{bmatrix} 0 & \alpha \\ 1 & 1 \end{bmatrix}$ and $0 < \alpha < 2$. If the condition number $k(A)$ of the matrix A is 6, then α equals to:

(A) 0.5 **(B)** 0.8 **(C)** 0.25 **(D)** None of These

Question 6: Let $p_1(x)$ the linear Lagrange interpolation polynomial for the data $(2, 4)$ and $(5, \alpha)$. If the constant term in $p_1(x)$ is equal 6, then the value of α is:

(A) -1 **(B)** 1 **(C)** 2 **(D)** None of These

Question 7: Let $x_0 = 2,\ x_1 = 2.5,\ x_2 = 4$ and $x_3 = 4.6$. If the best approximation of $f(x) = \frac{1}{x}$ at $x = 3$ using quadratic interpolation formula is $p_2(3) = 0.3250$, then the value of the unknown point η in the error formula is equal to:

(A) 2.9201 **(B)** 2.7859 **(C)** 3.1472 **(D)** None of These

Use the data in the following table to answer the Question 8 up to Question 11:

x	1.00	1.20	2.00	2.25	2.60	2.75	2.80	3.00
$f(x)$	0.4	1.5	2.03	2.7	3.7	4.5	5.2	α

Question 8: The best approximate value of $f'(2.5)$ using 3-point difference formula is:

(A) 3.6 **(B)** 2.6 **(C)** 1.6 **(D)** None of These

Question 9: If the best approximate value of $f'(3)$ using 3-point difference formula is 6, then the value of α is:

(A) 4.5 **(B)** 5.5 **(C)** 6.5 **(D)** None of These

Question 10: The best approximate value of $f''(2)$ using 3-point central difference formula is:

(A) 3.6600 **(B)** 1.6500 **(C)** 4.1250 **(D)** None of These

Question 11: If $\alpha = 6$, then the best approximate value of $\int_1^3 f(x)\, dx$ using Simpson's rule is:

(A) 4.84 **(B)** 5.23 **(C)** 6.72 **(D)** None of These

Question 12: The absolute error by using the Eluer's method for the approximation of $y(0.2)$ for the initial value problem $y' + 2y = e^{-2x}$, $y(0) = 0.1$, $n = 2$, and with the exact solution $y(x) = (0.1 + x)e^{-2x}$, is:

(A) 0.2259 **(B)** 0.0148 **(C)** 0.0248 **(D)** None of These

Question 13: By using the Taylor's method of order 2 for the initial value problem $xy' - x^2y = x^2$, $y(0) = 0$, $n = 2$, the approximation of $y(0.4)$ is:

(A) 0.0820 **(B)** 0.0828 **(C)** 0.0840 **(D)** None of These

<u>**Question 14:**</u> Let $f(x) = e^{-x}$ and $x_0 = 0, x_1 = 0, x_2 = 1, x_3 = 1$, find the approximation of $1/e^{0.5}$ by using the quadratic Newton's polynomial $p_2(x)$ and then use approximation by $p_2(x)$ to find the approximation of $1/e^{0.5}$ by using the cubic Newton's polynomial $p_3(x)$. Compute absolute error and the error bound for the approximation by cubic Newton's polynomial.

Question 15: Determine the number of subintervals n required to approximate the integral $\int_0^2 \frac{1}{x+4}dx$, with an error less than 10^{-4} using composite Simpson's rule. Then use the value of n to find the absolute errors in the approximations of the given integral by using Trapezoidal and Simpson's rules.

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Question 1: The first approximation of the square root of 19 using a quadratic convergent method with $x_0 = 5$ is:

(A) 4.36 **(B)** 4.44 **(C)** 4.40 **(D)** None of these

Question 2: The iterative scheme $x_{n+1} = 2 + (3+k)x_n - kx_n^3, \; n \geq 0$ converges at least quadratically to the root $\alpha = -1$ if k is:

(A) 1 (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) None of these

Question 3: Let $A = \begin{bmatrix} 1.001 & 1.5 \\ 2 & 3 \end{bmatrix}$, then the determinant of the lower-triangular matrix L of the LU factorization using Crout's method is:

(A) 0.003 (B) 1.001 (C) 2.002 (D) None of these

Question 4: The l_∞-norm of the first approximation of the solution vector obtained by Gauss-Seidel method starting with $\mathbf{x}^{(\mathbf{0})} = [0,0,0]^T$ for the following linear system $4x_1 - x_2 + x_3 = 4, \quad 3x_1 + 8x_2 + x_3 = -21, \quad -2x_1 + x_2 + 5x_3 = 15$; is:

(A) 3 **(B)** 4 **(C)** 8 **(D)** None of These

Question 5: Let $A = \begin{bmatrix} 0 & \alpha \\ 1 & 1 \end{bmatrix}$ and $0 < \alpha < 2$. If the condition number $k(A)$ of the matrix A is 6, then α equals to:

(A) 0.25 **(B)** 0.8 **(C)** 0.5 **(D)** None of These

Question 6: Let $p_1(x)$ the linear Lagrange interpolation polynomial for the data $(2,4)$ and $(5,\alpha)$. If the constant term in $p_1(x)$ is equal 6, then the value of α is:

(A) 1 **(B)** -1 **(C)** 2 **(D)** None of These

Question 7: Let $x_0 = 2$, $x_1 = 2.5$, $x_2 = 4$ and $x_3 = 4.6$. If the best approximation of $f(x) = \dfrac{1}{x}$ at $x = 3$ using quadratic interpolation formula is $p_2(3) = 0.3250$, then the value of the unknown point η in the error formula is equal to:

(A) 3.1472 **(B)** 2.9201 **(C)** 2.7859 **(D)** None of These

Use the data in the following table to answer the Question 8 up to Question 11:

x	1.00	1.20	2.00	2.25	2.60	2.75	2.80	3.00
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Question 8: The best approximate value of $f'(2.5)$ using 3-point difference formula is:

(A) 2.6 **(B)** 3.6 **(C)** 1.6 **(D)** None of These

Question 9: If the best approximate value of $f'(3)$ using 3-point difference formula is 6, then the value of α is:

(A) 5.5 **(B)** 6.5 **(C)** 4.5 **(D)** None of These

Question 10: The best approximate value of $f''(2)$ using 3-point central difference formula is:

(A) 4.1250 **(B)** 1.6500 **(C)** 3.6600 **(D)** None of These

Question 11: If $\alpha = 6$, then the best approximate value of $\int_1^3 f(x)\,dx$ using Simpson's rule is:

(A) 6.72 **(B)** 5.23 **(C)** 4.84 **(D)** None of These

Question 12: The absolute error by using the Eluer's method for the approximation of $y(0.2)$ for the initial value problem $y' + 2y = e^{-2x}$, $y(0) = 0.1$, $n = 2$, and with the exact solution $y(x) = (0.1 + x)e^{-2x}$, is:

(A) 0.0148 **(B)** 0.0248 **(C)** 0.2259 **(D)** None of These

Question 13: By using the Taylor's method of order 2 for the initial value problem $xy' - x^2y = x^2$, $y(0) = 0$, $n = 2$, the approximation of $y(0.4)$ is:

(A) 0.0840 **(B)** 0.0828 **(C)** 0.0820 **(D)** None of These

Question 14: Let $f(x) = e^{-x}$ and $x_0 = 0, x_1 = 0, x_2 = 1, x_3 = 1$, find the approximation of $1/e^{0.5}$ by using the quadratic Newton's polynomial $p_2(x)$ and then use approximation by $p_2(x)$ to find the approximation of $1/e^{0.5}$ by using the cubic Newton's polynomial $p_3(x)$. Compute absolute error and the error bound for the approximation by cubic Newton's polynomial.

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(A) 0.8 **(B)** 0.5 **(C)** 0.25 **(D)** None of These

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