KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Semester 461 / MATH-244 (Linear Algebra) / Mid-term Exam 2

Max. Marks: 25 **Max. Time: $1\frac{1}{2}$ hrs.**

Note: Scientific calculators are not allowed.

Question 1: [Marks: 1+1+1+1+1]

Which of the given choices are correct?

(i) Let $A = \{u_1, u_2, u_3, u_4\}$ is a subset of $\mathbb{R}^3$. Then the set A must be:

a) a subspace of $\mathbb{R}^3$ b) linearly dependent c) linearly independent d) a basis of $\mathbb{R}^3$.

(ii) For the matrix $M = \begin{bmatrix} 1 & -2 & 0 & 3 & -4 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$, which of the following statements is true?

a) $nullity(M) = 2$ b) $rank(M) = 3$ c) $nullity(M) = 3$ d) $rank(M) = 0$.

(iii) Let $B = \{(1,0,0,1), (-1,1,0,1), (0,0,1,1)\}$ and $C = \{v_1, v_2, v_3\}$ be two ordered bases for a vector subspace of the Euclidean space $\mathbb{R}^4$. If ${}_BP_C = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 2 & 0 & 0 \end{bmatrix}$ is the transition matrix from C to B, then the vector v_3 is equal to:

a) (1,1,0) b) (2,0,0) c) (2,0,0,2) d) (0,1,0,2).

(iv) Let u and v be any two vectors in a real inner product space $(V, <\,,\,>)$ such that $\|u\| = 3 = \|v\|$. Which of the following statements is true?

a) $|<u,v>| \leq 6$ b) $|<u,v>| > 6$ c) $|<u,v>| > 9$ d) $|<u,v>| \leq 9$.

(v) Let $W = \{w_1, w_2, w_3, w_4, w_5\}$ be an orthogonal set of nonzero vectors in an inner product space E of dimension 5. Then W must be:

a) a basis for E b) a subspace of E c) equal to E d) linearly dependent.

Question 2: [Marks: 3 + 3 + 4]

(a) Show that $F = \{(x, y, z) \in \mathbb{R}^3;\ x - y + z = 0,\ 2x + y - z = 0, x + y + z = 0\}$ is a vector subspace of $\mathbb{R}^3$.

(b) Show that $V = \{\begin{bmatrix} 0 & 2x - y \\ x & y \end{bmatrix} : x, y \in \mathbb{R}\}$ is a real vector space under usual addition and scalar multiplication of matrices. Also find $dim(V)$.

(c) Let $G = \{(0,0,3,3), (1,0,0,1), (-1,1,0,1), (0,0,1,1)\} \subseteq \mathbb{R}^4$. Find a basis B for $span(G)$ with $B \subseteq G$ and then find a basis C for $\mathbb{R}^4$ such that $B \subseteq C$.

Question 3: [Marks: 4 + 3 + 3]

(a) Let $B = \{(0,1,1), (1,1,0), (1,0,1)\}$ and $C = \{(1,1,0), (1,0,2), (1,1,1)\}$ be two ordered bases for the Euclidean space $\mathbb{R}^3$ and $[v]_C = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Then construct the transition matrix ${}_BP_C$ from basis C to B, and then find the coordinate vector $[v]_B$.

(b) Consider the vector space $M_2(\mathbb{R})$ of 2×2 real matrices with the inner product:

$$< A, B > := trace(AB^T),\ \forall\, A, B \in M_2(\mathbb{R}).$$

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then compute the angle θ between the matrices A and B.

(c) Let x and y be nonzero orthogonal vectors in an inner product space. Then show that $\{x, y\}$ is linearly independent and $\|x + y\|^2 = \|x\|^2 + \|y\|^2$.

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