KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Semester 461 / MATH-244 (Linear Algebra) / Mid-term Exam 1

Max. Marks: 25 **Max. Time: $1\frac{1}{2}$ hrs.**

Note: Scientific calculators are not allowed.

Question 1: [Marks: 5]

Determine whether the following statements are true or false:

(i) If A is a square matrix and $A^2 = 0$, then $(I + A)^{-1} = I - A$.

(ii) If A and B are row equivalent square matrices, then $|A| = |B|$.

(iii) If $A\,adj(A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $|A| = 9$.

(iv) There is a homogeneous linear equation for which (1, 0, 2) is a solution but not (2, 0, –4).

(v) If $RREF(A)$ has a zero row, then $AX \doteq B$ must have infinitely many solutions.

Question 2: [Marks: 4 + 3 + 3]

(a) Find inverse of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ ~~as a product of elementary matrices~~ and find the matrix B satisfying the equation $BA = A^2 + 5A$.

(b) Let A be an invertible matrix. Show that $(adj(A))^{-1} = adj(A^{-1})$.

(c) Show that the matrix $\begin{bmatrix} 1 & 0 & 1+x \\ 1 & 1 & 0 \\ -x & 0 & 1 \end{bmatrix}$ is invertible for all $x \varepsilon \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers.

Question 3: [Marks: 2 + 3 + 5]

(a) If E is an elementary matrix, then show that the linear system $EX = O$ has only the trivial solution.

(b) Solve the following system of linear equations:

$$\begin{aligned} x + y &= 1 \\ x + 2y + z &= -1 \\ x + 3y - z &= 2. \end{aligned}$$

(c) What conditions must a, b, c, and d satisfy for the following system to be consistent?

$$\begin{aligned} x_1 + x_2 - x_4 &= a \\ x_2 - x_3 - 2x_4 &= b \\ 2x_1 + 2x_3 + 2x_4 &= c \\ 2x_1 + x_2 + x_3 &= d. \end{aligned}$$

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