KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Semester 462 / MATH-244 (Linear Algebra) / Make-up of Mid-term Exams
Max. Marks: 25 **Max. Time: 3 hrs.**

Note: Calculators are not allowed.

Question 1: [Marks: 1+1+1+1+1]

Which of the given choices are correct?

(i) If A is a square matrix satisfying $A^2 = O$, then $(I-2A)^{-1}$ is equal to:

a) $I-2A$ b) $2A-I$ c) $I+2A$ d) $2I+A$.

(*ii*) If $adj(A)A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then the determinant $|A|$ is equal to:

a) 3 b) 1 c) 9 d) 6.

(iii) Let $B = \{u_1, u_2, u_3, u_4, u_5\}$ is a linearly independent subset of $\mathbb{R}^5$. Then the set B must be:

a) a vector subspace of $\mathbb{R}^5$ b) the standard basis of $\mathbb{R}^5$ c) a basis of $\mathbb{R}^4$ d) a basis of $\mathbb{R}^5$.

(iv) Let ${}_C P_B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$ be the transition matrix from ordered basis $B = \{u_1, u_2, u_3\}$ to ordered basis C of a vector space V. Then, the coordinate vector $[u_1 - 2u_3]_C$ is equal to:

a) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ c) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ d) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

(v) If $\{(-3r+4s, r-s, r, s) \mid r, s \in \mathbb{R}\}$ is the solution space of homogeneous system $AX = 0$, then $rank(A)$ is equal to:

a) 1 b) 2 c) 3 d) 4

Question 2: [Marks: 4 + 2 + 4]

a) Find inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ by using $adj(A)$.

b) Let B be a 3×3 matrix with $|B| = 2$. Evaluate the determinant $|adj(adj(B))|$.

c) Find leading and free variables in the following system of equations and then find its solution set:

$$\begin{aligned} 2w - x - 4y + 3z &= 1 \\ 3w - 2x - 5y + 4z &= 1 \\ 3w - 3x - 2y \quad &= 1 \end{aligned}$$

Question 3: [Marks: 3 + 3 + 4]

a) Find a basis B of the vector space $\mathbb{R}^4$ such that $B \supseteq \{u_1 = (1, 2, 3, 0), u_2 = (1, 2, 0, 0), u_3 = (1, 0, 0, 1)\}$.

b) Find nullity and rank of the matrix $\begin{bmatrix} 1 & -1 & 0 & -1 \\ -1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix}$.

c) Let $C = \{c_1 = (1,1,0), c_2 = (1,0,2), c_3 = (0,1,1)\}$ and $D = \{d_1 = (0,1,2), d_2 = (1,0,0), d_3 = (0,0,1)\}$ be two ordered bases for the vector space $\mathbb{R}^3$ and $u \in \mathbb{R}^3$ with $[u]_D = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. Find the vector u, the transition matrix ${}_C P_D$ from basis D to C, and the coordinate vector $[u]_C$.

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