جامعة الملك سعود
King Saud University

كلية العلوم
قسم الرياضيات

Math 243 Final Exam

Second Semester 1447H

Please follow the following instructions:
1- Write your answer in the same order as the questions.
2- Write legibly.

Question 1: (5 points)
Find the primitive and non-primitive triples (x, y, z), such that $x = 35$.

Question 2: (5 points)
Find all x such that $\varphi(x) = 4$, where φ is Euler's function.

Question 3: (5 points)
Define the Möbius function μ, then prove that it is multiplicative.

Question 4: (5 points)
Let $p \geq 2$, p is prime if and only if $(p-1)! \equiv -1 \ (mod\ p)$. (Wilson's Theorem)

Question 5: (2+3=5 points)
(a) Define Carmichael numbers.
(b) Give an example of a Carmichael number with justification.

Question 6: (2+3=5 points)
(a) state the theorem that give a necessary and sufficient condition for an even number n to be perfect number.
(b) Use the formula in (a) to find four even perfect numbers.

Question 7: (5 points)
If $ca \equiv cb \ (mod\ n)$, then prove that $a \equiv b \left(mod\ \frac{n}{d}\right)$, where $d = (c, n)$. Is the converse true? Justify your answer.

Question 8: (2+3=5 points)
(a) Define a pseudo-prime to the base b.
(b) Give an example of a pseudo-prime to the base 2 with justification.

Extra Question for bonus: (5 points)

1- Let $n = 2100$, find the following:
   a- $\varphi(n)$.
   b- $\sigma(n)$.
   c- $\tau(n)$.
   d- $\mu(n)$.
   e- Is n a perfect number (justify your answer).

Good Luck