SOCIETY OF ACTUARIES

EXAM P   PROBABILITY

EXAM P SAMPLE QUESTIONS

Some of the questions in this study note are taken from past examinations.

Some of the questions have been reformatted from previous versions of this note.

Questions 154 and 155 were added in October 2014.

Questions 156 – 206 were added January 2015.

Questions 207 – 237 were added April 2015.

Questions 238-240 were added May 2015.

Questions 241-242 were added November 2015.

1.   A survey of a group's viewing habits over the last year revealed the following information:

> (i)   28% watched gymnastics
> (ii)   29% watched baseball
> (iii)   19% watched soccer
> (iv)   14% watched gymnastics and baseball
> (v)   12% watched baseball and soccer
> (vi)   10% watched gymnastics and soccer
> (vii)   8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

(A)   24%
(B)   36%
(C)   41%
(D)   52%
(E)   60%

2.   The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work.

Calculate the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

(A)   0.05
(B)   0.12
(C)   0.18
(D)   0.25
(E)   0.35

3.   You are given $P[A \cup B] = 0.7$ and $P[A \cup B'] = 0.9$.

Calculate $P[A]$.

(A)   0.2
(B)   0.3
(C)   0.4
(D)   0.6
(E)   0.8

4. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44.

Calculate the number of blue balls in the second urn.

(A) 4
(B) 20
(C) 24
(D) 44
(E) 64

5. An auto insurance company has 10,000 policyholders. Each policyholder is classified as

      (i)   young or old;
      (ii)   male or female; and
      (iii)   married or single.

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males.

Calculate the number of the company's policyholders who are young, female, and single.

(A) 280
(B) 423
(C) 486
(D) 880
(E) 896

6. A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease.

Calculate the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

(A) 0.115
(B) 0.173
(C) 0.224
(D) 0.327
(E) 0.514

7.   An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto policy and a homeowners policy will renew at least one of those policies next year.

Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto policy and a homeowners policy.

Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

(A)   20%
(B)   29%
(C)   41%
(D)   53%
(E)   70%

8.   Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist.

Calculate the probability that a randomly chosen member of this group visits a physical therapist.

(A)   0.26
(B)   0.38
(C)   0.40
(D)   0.48
(E)   0.62

9. An insurance company examines its pool of auto insurance customers and gathers the following information:

> (i) All customers insure at least one car.
> (ii) 70% of the customers insure more than one car.
> (iii) 20% of the customers insure a sports car.
> (iv) Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

> (A) 0.13
> (B) 0.21
> (C) 0.24
> (D) 0.25
> (E) 0.30

10. Question duplicates Question 9 and has been deleted.

11. An actuary studying the insurance preferences of automobile owners makes the following conclusions:

> (i) An automobile owner is twice as likely to purchase collision coverage as disability coverage.
> (ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.
> (iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15.

Calculate the probability that an automobile owner purchases neither collision nor disability coverage.

> (A) 0.18
> (B) 0.33
> (C) 0.48
> (D) 0.67
> (E) 0.82

12. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

> (i)   14% have high blood pressure.
> (ii)   22% have low blood pressure.
> (iii)   15% have an irregular heartbeat.
> (iv)   Of those with an irregular heartbeat, one-third have high blood pressure.
> (v)   Of those with normal blood pressure, one-eighth have an irregular heartbeat.

Calculate the portion of the patients selected who have a regular heartbeat and low blood pressure.

(A)   2%
(B)   5%
(C)   8%
(D)   9%
(E)   20%

13. An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is 1/3.

Calculate the probability that a woman has none of the three risk factors, given that she does not have risk factor A.

(A)   0.280
(B)   0.311
(C)   0.467
(D)   0.484
(E)   0.700

14. In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $p(n+1) = 0.2 p(n)$ where $p(n)$ represents the probability that the policyholder files n claims during the period.

Under this assumption, calculate the probability that a policyholder files more than one claim during the period.

(A) 0.04
(B) 0.16
(C) 0.20
(D) 0.80
(E) 0.96

15. An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are 1/4, 1/3, and 5/12 respectively.

Calculate the probability that a randomly chosen employee will choose no supplementary coverage.

(A) 0
(B) 47/144
(C) 1/2
(D) 97/144
(E) 7/9

16. An insurance company determines that N, the number of claims received in a week, is a random variable with $P[N = n] = \dfrac{1}{2^{n+1}}$ where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week.

Calculate the probability that exactly seven claims will be received during a given two-week period.

(A) 1/256
(B) 1/128
(C) 7/512
(D) 1/64
(E) 1/32

17. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

Calculate the probability that a claim submitted to the insurance company includes operating room charges.

(A)   0.10
(B)   0.20
(C)   0.25
(D)   0.40
(E)   0.80

18. Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.0056h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.0044h$.

The errors from the two instruments are independent of each other.

Calculate the probability that the average value of the two measurements is within $0.005h$ of the height of the tower.

(A)   0.38
(B)   0.47
(C)   0.68
(D)   0.84
(E)   0.90

19.   An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident.

Calculate the probability that the driver was age 16-20.

(A)   0.13
(B)   0.16
(C)   0.19
(D)   0.23
(E)   0.40

20.   An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the company's policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policyholder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year.

A policyholder dies in the next year.

Calculate the probability that the deceased policyholder was ultra-preferred.

(A)   0.0001
(B)   0.0010
(C)   0.0071
(D)   0.0141
(E)   0.2817

21. Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

   (i)   10% of the emergency room patients were critical;
   (ii)   30% of the emergency room patients were serious;
   (iii)   the rest of the emergency room patients were stable;
   (iv)   40% of the critical patients died;
   (vi)   10% of the serious patients died; and
   (vii)   1% of the stable patients died.

Given that a patient survived, calculate the probability that the patient was categorized as serious upon arrival.

   (A)   0.06
   (B)   0.29
   (C)   0.30
   (D)   0.39
   (E)   0.64

22. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died during the five-year period.

Calculate the probability that the participant was a heavy smoker.

   (A)   0.20
   (B)   0.25
   (C)   0.35
   (D)   0.42
   (E)   0.57

23.   An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are:

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

Given that a driver has been involved in at least one collision in the past year, calculate the probability that the driver is a young adult driver.

(A)   0.06
(B)   0.16
(C)   0.19
(D)   0.22
(E)   0.25

24.   The number of injury claims per month is modeled by a random variable N with

$$P[N=n]=\frac{1}{(n+1)(n+2)}, \text{ for nonnegative integers, } n.$$

Calculate the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

(A)   1/3
(B)   2/5
(C)   1/2
(D)   3/5
(E)   5/6

25.   A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not actually present. One percent of the population actually has the disease.

Calculate the probability that a person actually has the disease given that the test indicates the presence of the disease.

(A)   0.324
(B)   0.657
(C)   0.945
(D)   0.950
(E)   0.995

26.   The probability that a randomly chosen male has a blood circulation problem is 0.25. Males who have a blood circulation problem are twice as likely to be smokers as those who do not have a blood circulation problem.

Calculate the probability that a male has a blood circulation problem, given that he is a smoker.

(A)   1/4
(B)   1/3
(C)   2/5
(D)   1/2
(E)   2/3

27. A study of automobile accidents produced the following data:

Model year	Proportion of all vehicles	Probability of involvement in an accident
2014	0.16	0.05
2013	0.18	0.02
2012	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 2014, 2013, and 2012 was involved in an accident.

Calculate the probability that the model year of this automobile is 2014.

(A)   0.22
(B)   0.30
(C)   0.33
(D)   0.45
(E)   0.50

28. A hospital receives 1/5 of its flu vaccine shipments from Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials.

For Company X's shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective.

Calculate the probability that this shipment came from Company X.

(A)   0.10
(B)   0.14
(C)   0.37
(D)   0.63
(E)   0.86

29. The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year.

Calculate the portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year.

(A)   0.15
(B)   0.34
(C)   0.43
(D)   0.57
(E)   0.66

30. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims.

The number of claims filed has a Poisson distribution.

Calculate the variance of the number of claims filed.

(A) $\dfrac{1}{\sqrt{3}}$

(B)   1

(C) $\sqrt{2}$

(D)   2

(E)   4

31. A company establishes a fund of 120 from which it wants to pay an amount, C, to any of its 20 employees who achieve a high performance level during the coming year. Each employee has a 2% chance of achieving a high performance level during the coming year. The events of different employees achieving a high performance level during the coming year are mutually independent.

Calculate the maximum value of C for which the probability is less than 1% that the fund will be inadequate to cover all payments for high performance.

(A)   24
(B)   30
(C)   40
(D)   60
(E)   120

32. A large pool of adults earning their first driver's license includes 50% low-risk drivers, 30% moderate-risk drivers, and 20% high-risk drivers. Because these drivers have no prior driving record, an insurance company considers each driver to be randomly selected from the pool.

This month, the insurance company writes four new policies for adults earning their first driver's license.

Calculate the probability that these four will contain at least two more high-risk drivers than low-risk drivers.

(A)   0.006
(B)   0.012
(C)   0.018
(D)   0.049
(E)   0.073

33. The loss due to a fire in a commercial building is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20 - x), & 0 < x < 20 \\ 0, & \text{otherwise.} \end{cases}$$

Given that a fire loss exceeds 8, calculate the probability that it exceeds 16.

(A)   1/25
(B)   1/9
(C)   1/8
(D)   1/3
(E)   3/7

34. The lifetime of a machine part has a continuous distribution on the interval (0, 40) with probability density function $f(x)$, where $f(x)$ is proportional to $(10 + x)^{-2}$ on the interval.

Calculate the probability that the lifetime of the machine part is less than 6.

(A)   0.04
(B)   0.15
(C)   0.47
(D)   0.53
(E)   0.94

35. This question duplicates Question 34 and has been deleted.

36. A group insurance policy covers the medical claims of the employees of a small company. The value, V, of the claims made in one year is described by

$$V = 100{,}000Y$$

where Y is a random variable with density function

$$f(y) = \begin{cases} k(1-y)^4, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

Calculate the conditional probability that V exceeds 40,000, given that V exceeds 10,000.

(A) 0.08
(B) 0.13
(C) 0.17
(D) 0.20
(E) 0.51

37. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, a one-half refund if it fails during the second year, and no refund for failure after the second year.

Calculate the expected total amount of refunds from the sale of 100 printers.

(A) 6,321
(B) 7,358
(C) 7,869
(D) 10,256
(E) 12,642

38. An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & \text{otherwise.} \end{cases}$$

Given that a randomly selected home is insured for at least 1.5, calcuate the probability that it is insured for less than 2.

(A)   0.578
(B)   0.684
(C)   0.704
(D)   0.829
(E)   0.875

39. A company prices its hurricane insurance using the following assumptions:

(i)   In any calendar year, there can be at most one hurricane.
(ii)   In any calendar year, the probability of a hurricane is 0.05.
(iii) The numbers of hurricanes in different calendar years are mutually independent.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(A)   0.06
(B)   0.19
(C)   0.38
(D)   0.62
(E)   0.92

40. An insurance policy pays for a random loss X subject to a deductible of C, where $0 < C < 1$. The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Given a random loss X, the probability that the insurance payment is less than 0.5 is equal to 0.64.

Calculate C.

(A)   0.1
(B)   0.3
(C)   0.4
(D)   0.6
(E)   0.8

41. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants).

Calculate the probability that at least nine participants complete the study in one of the two groups, but not in both groups?

(A)   0.096
(B)   0.192
(C)   0.235
(D)   0.376
(E)   0.469

42. For Company A there is a 60% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 10,000 and standard deviation 2,000.

For Company B there is a 70% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 9,000 and standard deviation 2,000.

The total claim amounts of the two companies are independent.

Calculate the probability that, in the coming year, Company B's total claim amount will exceed Company A's total claim amount.

(A) 0.180
(B) 0.185
(C) 0.217
(D) 0.223
(E) 0.240

43. A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 0.60. The numbers of accidents that occur in different months are mutually independent.

Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs.

(A) 0.01
(B) 0.12
(C) 0.23
(D) 0.29
(E) 0.41

44. An insurance policy pays 100 per day for up to three days of hospitalization and 50 per day for each day of hospitalization thereafter.

The number of days of hospitalization, X, is a discrete random variable with probability function

$$P[X = k] = \begin{cases} \dfrac{6-k}{15}, & k = 1,2,3,4,5 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the expected payment for hospitalization under this policy.

(A)   123
(B)   210
(C)   220
(D)   270
(E)   367

45. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \dfrac{|x|}{10}, & -2 \le x \le 4 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of X.

(A)   1/5
(B)   3/5
(C)   1
(D)   28/15
(E)   12/5

46.   A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$.

Calculate $E(X)$.

(A) $2 + \dfrac{1}{3}e^{-6}$
(B) $2 - 2e^{-2/3} + 5e^{-4/3}$
(C)   3
(D) $2 + 3e^{-2/3}$
(E)   5

47.   A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made.

Calculate x such that the expected payment made under this insurance is 1000.

(A)   3858
(B)   4449
(C)   5382
(D)   5644
(E)   7235

48.   An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4.

Calculate the expected benefit under this policy.

(A)   2234
(B)   2400
(C)   2500
(D)   2667
(E)   2694

49. This question duplicates Question 44 and has been deleted

50. A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5.

Calculate the expected amount paid to the company under this policy during a one-year period.

(A)   2,769
(B)   5,000
(C)   7,231
(D)   8,347
(E)   10,578

51. A manufacturer's annual losses follow a distribution with density function

$$f(x) = \begin{cases} \dfrac{2.5(0.6)^{2.5}}{x^{3.5}}, & x > 0.6 \\ 0, & \text{otherwise.} \end{cases}$$

To cover its losses, the manufacturer purchases an insurance policy with an annual deductible of 2.

Calculate the mean of the manufacturer's annual losses not paid by the insurance policy.

(A)   0.84
(B)   0.88
(C)   0.93
(D)   0.95
(E)   1.00

52. An insurance company sells a one-year automobile policy with a deductible of 2. The probability that the insured will incur a loss is 0.05. If there is a loss, the probability of a loss of amount N is K/N, for $N = 1, \ldots, 5$ and K a constant. These are the only possible loss amounts and no more than one loss can occur.

Calculate the expected payment for this policy.

(A) 0.031
(B) 0.066
(C) 0.072
(D) 0.110
(E) 0.150

53. An insurance policy reimburses a loss up to a benefit limit of 10. The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} 2y^{-3}, & y > 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of the benefit paid under the insurance policy.

(A) 1.0
(B) 1.3
(C) 1.8
(D) 1.9
(E) 2.0

54. An auto insurance company insures an automobile worth 15,000 for one year under a policy with a 1,000 deductible. During the policy year there is a 0.04 chance of partial damage to the car and a 0.02 chance of a total loss of the car. If there is partial damage to the car, the amount X of damage (in thousands) follows a distribution with density function

$$f(x) = \begin{cases} 0.5003e^{-x/2}, & 0 < x < 15 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected claim payment.

(A) 320
(B) 328
(C) 352
(D) 380
(E) 540

55. An insurance company's monthly claims are modeled by a continuous, positive random variable X, whose probability density function is proportional to $(1 + x)^{-4}$, for $0 < x < \infty$. Calculate the company's expected monthly claims.

(A)   1/6
(B)   1/3
(C)   1/2
(D)   1
(E)   3

56.   An insurance policy is written to cover a loss, X, where X has a uniform distribution on [0, 1000]. The policy has a deductible, d, and the expected payment under the policy is 25% of what it would be with no deductible.

Calculate d.

(A)   250
(B)   375
(C)   500
(D)   625
(E)   750

57.   An actuary determines that the claim size for a certain class of accidents is a random variable, X, with moment generating function

$$M_X(t) = \frac{1}{(1-2500t)^4} .$$

Calculate the standard deviation of the claim size for this class of accidents.

(A)   1,340
(B)   5,000
(C)   8,660
(D)   10,000
(E)   11,180

58. A company insures homes in three cities, J, K, and L. Since sufficient distance separates the cities, it is reasonable to assume that the losses occurring in these cities are mutually independent.

The moment generating functions for the loss distributions of the cities are:

$$M_J(t) = (1-2t)^{-3}, \quad M_K(t) = (1-2t)^{-2.5}, \quad M_L(t) = (1-2t)^{-4.5}.$$

Let X represent the combined losses from the three cities.

Calculate $E(X^3)$.

(A) 1,320
(B) 2,082
(C) 5,760
(D) 8,000
(E) 10,560

59. An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x) = \begin{cases} \dfrac{2.5(200)^{2.5}}{x^{3.5}}, & x > 200 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the difference between the 30th and 70th percentiles of X.

(A) 35
(B) 93
(C) 124
(D) 231
(E) 298

60. A recent study indicates that the annual cost of maintaining and repairing a car in a town in Ontario averages 200 with a variance of 260.

A tax of 20% is introduced on all items associated with the maintenance and repair of cars (i.e., everything is made 20% more expensive).

Calculate the variance of the annual cost of maintaining and repairing a car after the tax is introduced.

(A)   208
(B)   260
(C)   270
(D)   312
(E)   374

61. This question duplicates Question 59 and has been deleted

62. A random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 1 \\ \dfrac{x^2 - 2x + 2}{2}, & 1 \le x < 2 \\ 1, & x \ge 2. \end{cases}$$

Calculate the variance of X.

(A)   7/72
(B)   1/8
(C)   5/36
(D)   4/3
(E)   23/12

63. The warranty on a machine specifies that it will be replaced at failure or age 4, whichever occurs first. The machine's age at failure, X, has density function

$$f(x) = \begin{cases} 1/5, & 0 < x < 5 \\ 0, & \text{otherwise.} \end{cases}$$

Let Y be the age of the machine at the time of replacement.

Calculate the variance of Y.

(A) 1.3
(B) 1.4
(C) 1.7
(D) 2.1
(E) 7.5

64. A probability distribution of the claim sizes for an auto insurance policy is given in the table below:

Claim Size	Probability
20	0.15
30	0.10
40	0.05
50	0.20
60	0.10
70	0.10
80	0.30

Calculate the percentage of claims that are within one standard deviation of the mean claim size.

(A) 45%
(B) 55%
(C) 68%
(D) 85%
(E) 100%

65. The owner of an automobile insures it against damage by purchasing an insurance policy with a deductible of 250. In the event that the automobile is damaged, repair costs can be modeled by a uniform random variable on the interval (0, 1500).

Calculate the standard deviation of the insurance payment in the event that the automobile is damaged.

(A) 361
(B) 403

(C)   433
(D)   464
(E)   521

66.   DELETED

67.   A baseball team has scheduled its opening game for April 1. If it rains on April 1, the game is postponed and will be played on the next day that it does not rain. The team purchases insurance against rain. The policy will pay 1000 for each day, up to 2 days, that the opening game is postponed.

The insurance company determines that the number of consecutive days of rain beginning on April 1 is a Poisson random variable with mean 0.6.

Calculate the standard deviation of the amount the insurance company will have to pay.

(A)   668
(B)   699
(C)   775
(D)   817
(E)   904

68. An insurance policy reimburses dental expense, X, up to a maximum benefit of 250. The probability density function for X is:

$$f(x) = \begin{cases} ce^{-0.004x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where c is a constant.

Calculate the median benefit for this policy.

(A)   161
(B)   165
(C)   173
(D)   182
(E)   250

69. The time to failure of a component in an electronic device has an exponential distribution with a median of four hours.

Calculate the probability that the component will work without failing for at least five hours.

(A)   0.07
(B)   0.29
(C)   0.38
(D)   0.42
(E)   0.57

70. An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100. Losses incurred follow an exponential distribution with mean 300.

Calculate the 95$^{\text{th}}$ percentile of losses that exceed the deductible.

(A)   600
(B)   700
(C)   800
(D)   900
(E)   1000

71. The time, T, that a manufacturing system is out of operation has cumulative distribution function

$$F(t) = \begin{cases} 1 - \left(\dfrac{2}{t}\right)^2, & t > 2 \\ 0, & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = T^2$. Let g be the density function for Y.

Determine $g(y)$, for $y > 4$.

(A) $\dfrac{4}{y^2}$

(B) $\dfrac{8}{y^{3/2}}$

(C) $\dfrac{8}{y^3}$

(D) $\dfrac{16}{y}$

(E) $\dfrac{1024}{y^5}$

72. An investment account earns an annual interest rate R that follows a uniform distribution on the interval (0.04, 0.08). The value of a 10,000 initial investment in this account after one year is given by $V = 10,000e^R$.

Let F be the cumulative distribution function of V.

Determine $F(v)$ for values of v that satisfy $0 < F(v) < 1$.

(A) $\dfrac{10,000e^{v/10,000} - 10,408}{425}$

(B) $25e^{v/10,000} - 0.04$

(C) $\dfrac{v - 10,408}{10,833 - 10,408}$

(D) $\dfrac{25}{v}$

(E) $25\left[\ln\left(\dfrac{v}{10,000}\right) - 0.04\right]$

73. An actuary models the lifetime of a device using the random variable $Y = 10X^{0.8}$, where X is an exponential random variable with mean 1.

Let $f(y)$ be the density function for Y.

Determine $f(y)$, for $y > 0$.

(A) $10y^{0.8} \exp(-8y^{-0.2})$

(B) $8y^{-0.2} \exp(-10y^{0.8})$

(C) $8y^{-0.2} \exp[-(0.1y)^{1.25}]$

(D) $(0.1y)^{1.25} \exp[-0.125(0.1y)^{0.25}]$

(E) $0.125(0.1y)^{0.25} \exp[-(0.1y)^{1.25}]$

74. Let T denote the time in minutes for a customer service representative to respond to 10 telephone inquiries. T is uniformly distributed on the interval with endpoints 8 minutes and 12 minutes.

Let R denote the average rate, in customers per minute, at which the representative responds to inquiries, and let $f(r)$ be the density function for R.

Determine $f(r)$, for $\dfrac{10}{12} \leq r \leq \dfrac{10}{8}$.

(A) $\dfrac{12}{5}$

(B) $3 - \dfrac{5}{2r}$

(C) $3r - \dfrac{5\ln(r)}{2}$

(D) $\dfrac{10}{r^2}$

(E) $\dfrac{5}{2r^2}$

75. The monthly profit of Company I can be modeled by a continuous random variable with density function f. Company II has a monthly profit that is twice that of Company I.

Let g be the density function for the distribution of the monthly profit of Company II.

Determine $g(x)$ where it is not zero.

(A) $\dfrac{1}{2} f\left(\dfrac{x}{2}\right)$

(B) $f\left(\dfrac{x}{2}\right)$

(C) $2 f\left(\dfrac{x}{2}\right)$

(D) $2 f(x)$

(E) $2 f(2x)$

76. Claim amounts for wind damage to insured homes are mutually independent random variables with common density function

$$f(x) = \begin{cases} \dfrac{3}{x^4}, & x > 1 \\ 0, & \text{otherwise,} \end{cases}$$

where x is the amount of a claim in thousands.

Suppose 3 such claims will be made.

Calculate the expected value of the largest of the three claims.

(A) 2025
(B) 2700
(C) 3232
(D) 3375
(E) 4500

77. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x, y) = \frac{x+y}{8}, \text{ for } 0 < x < 2 \text{ and } 0 < y < 2.$$

Calculate the probability that the device fails during its first hour of operation.

(A)   0.125
(B)   0.141
(C)   0.391
(D)   0.625
(E)   0.875

78. This question duplicates Question 77 and has been deleted

79. A device contains two components. The device fails if either component fails. The joint density function of the lifetimes of the components, measured in hours, is $f(s,t)$, where $0 < s < 1$ and $0 < t < 1$.

Determine which of the following represents the probability that the device fails during the first half hour of operation.

(A) $\int_0^{0.5} \int_0^{0.5} f(s,t)dsdt$

(B) $\int_0^1 \int_0^{0.5} f(s,t)dsdt$

(C) $\int_{0.5}^1 \int_{0.5}^1 f(s,t)dsdt$

(D) $\int_0^{0.5} \int_0^1 f(s,t)dsdt + \int_0^1 \int_0^{0.5} f(s,t)dsdt$

(E) $\int_0^{0.5} \int_{0.5}^1 f(s,t)dsdt + \int_0^1 \int_0^{0.5} f(s,t)dsdt$

80. A charity receives 2025 contributions. Contributions are assumed to be mutually independent and identically distributed with mean 3125 and standard deviation 250.

Calculate the approximate 90th percentile for the distribution of the total contributions received.

(A) 6,328,000
(B) 6,338,000
(C) 6,343,000
(D) 6,784,000
(E) 6,977,000

81. Claims filed under auto insurance policies follow a normal distribution with mean 19,400 and standard deviation 5,000.

Calculate the probability that the average of 25 randomly selected claims exceeds 20,000.

(A) 0.01
(B) 0.15
(C) 0.27
(D) 0.33
(E) 0.45

82. An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policyholder under a vision care insurance policy during one year is a Poisson random variable with mean 2. Assume the numbers of claims filed by different policyholders are mutually independent.

Calculate the approximate probability that there is a total of between 2450 and 2600 claims during a one-year period?

(A) 0.68
(B) 0.82
(C) 0.87
(D) 0.95
(E) 1.00

83. A company manufactures a brand of light bulb with a lifetime in months that is normally distributed with mean 3 and variance 1. A consumer buys a number of these bulbs with the intention of replacing them successively as they burn out. The light bulbs have mutually independent lifetimes.

Calculate the smallest number of bulbs to be purchased so that the succession of light bulbs produces light for at least 40 months with probability at least 0.9772.

(A) 14
(B) 16
(C) 20
(D) 40
(E) 55

84. Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a three-month period. The following information is known about X and Y:

$E(X) = 50$, $E(Y) = 20$, $\text{Var}(X) = 50$, $\text{Var}(Y) = 30$, $\text{Cov}(X,Y) = 10$.

The totals of hours that different individuals watch movies and sporting events during the three months are mutually independent.

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these one hundred people watch movies or sporting events during this three-month period.

Approximate the value of $P[T < 7100]$.

(A) 0.62
(B) 0.84
(C) 0.87
(D) 0.92
(E) 0.97

85. The total claim amount for a health insurance policy follows a distribution with density function

$$f(x) = \frac{1}{1000} e^{-(x/1000)}, \ x > 0.$$

The premium for the policy is set at the expected total claim amount plus 100.

If 100 policies are sold, calculate the approximate probability that the insurance company will have claims exceeding the premiums collected.

(A) 0.001
(B) 0.159
(C) 0.333
(D) 0.407
(E) 0.460

86. A city has just added 100 new female recruits to its police force. The city will provide a pension to each new hire who remains with the force until retirement. In addition, if the new hire is married at the time of her retirement, a second pension will be provided for her husband. A consulting actuary makes the following assumptions:

(i) Each new recruit has a 0.4 probability of remaining with the police force until retirement.

(ii) Given that a new recruit reaches retirement with the police force, the probability that she is not married at the time of retirement is 0.25.

(iii) The events of different new hires reaching retirement and the events of different new hires being married at retirement are all mutually independent events.

Calculate the probability that the city will provide at most 90 pensions to the 100 new hires and their husbands.

(A) 0.60
(B) 0.67
(C) 0.75
(D) 0.93
(E) 0.99

87. In an analysis of healthcare data, ages have been rounded to the nearest multiple of 5 years. The difference between the true age and the rounded age is assumed to be uniformly distributed on the interval from -2.5 years to 2.5 years. The healthcare data are based on a random sample of 48 people.

Calculate the approximate probability that the mean of the rounded ages is within 0.25 years of the mean of the true ages.

(A)   0.14
(B)   0.38
(C)   0.57
(D)   0.77
(E)   0.88

88. The waiting time for the first claim from a good driver and the waiting time for the first claim from a bad driver are independent and follow exponential distributions with means 6 years and 3 years, respectively.

Calculate the probability that the first claim from a good driver will be filed within 3 years and the first claim from a bad driver will be filed within 2 years.

(A) $\dfrac{1}{18}\left(1-e^{-2/3}-e^{-1/2}+e^{-7/6}\right)$

(B) $\dfrac{1}{18}e^{-7/6}$

(C) $1-e^{-2/3}-e^{-1/2}+e^{-7/6}$

(D) $1-e^{-2/3}-e^{-1/2}+e^{-1/3}$

(E) $1-\dfrac{1}{3}e^{-2/3}-\dfrac{1}{6}e^{-1/2}+\dfrac{1}{18}e^{-7/6}$

89. The future lifetimes (in months) of two components of a machine have the following joint density function:

$$f(x, y) = \begin{cases} \dfrac{6}{125,000}(50 - x - y), & 0 < x < 50 - y < 50 \\ 0, & \text{otherwise.} \end{cases}$$

Determine which of the following represents the probability that both components are still functioning 20 months from now.

(A) $\dfrac{6}{125,000} \displaystyle\int_0^{20} \int_0^{20} (50 - x - y)\,dy\,dx$

(B) $\dfrac{6}{125,000} \displaystyle\int_{20}^{30} \int_{20}^{50-x} (50 - x - y)\,dy\,dx$

(C) $\dfrac{6}{125,000} \displaystyle\int_{20}^{30} \int_{20}^{50-x-y} (50 - x - y)\,dy\,dx$

(D) $\dfrac{6}{125,000} \displaystyle\int_{20}^{50} \int_{20}^{50-x} (50 - x - y)\,dy\,dx$

(E) $\dfrac{6}{125,000} \displaystyle\int_{20}^{50} \int_{20}^{50-x-y} (50 - x - y)\,dy\,dx$

90. An insurance company sells two types of auto insurance policies: Basic and Deluxe. The time until the next Basic Policy claim is an exponential random variable with mean two days. The time until the next Deluxe Policy claim is an independent exponential random variable with mean three days.

Calculate the probability that the next claim will be a Deluxe Policy claim.

(A) 0.172
(B) 0.223
(C) 0.400
(D) 0.487
(E) 0.500

91. An insurance company insures a large number of drivers. Let X be the random variable representing the company's losses under collision insurance, and let Y represent the company's losses under liability insurance. X and Y have joint density function

$$f(x, y) = \begin{cases} \dfrac{2x+2-y}{4}, & 0 < x < 1 \text{ and } 0 < y < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the probability that the total company loss is at least 1.

(A) 0.33
(B) 0.38
(C) 0.41
(D) 0.71
(E) 0.75

92. Two insurers provide bids on an insurance policy to a large company. The bids must be between 2000 and 2200. The company decides to accept the lower bid if the two bids differ by 20 or more. Otherwise, the company will consider the two bids further. Assume that the two bids are independent and are both uniformly distributed on the interval from 2000 to 2200.

Calculate the probability that the company considers the two bids further.

(A) 0.10
(B) 0.19
(C) 0.20
(D) 0.41
(E) 0.60

93. A family buys two policies from the same insurance company. Losses under the two policies are independent and have continuous uniform distributions on the interval from 0 to 10. One policy has a deductible of 1 and the other has a deductible of 2. The family experiences exactly one loss under each policy.

Calculate the probability that the total benefit paid to the family does not exceed 5.

(A) 0.13
(B) 0.25
(C) 0.30
(D) 0.32
(E) 0.42

94. Let T_1 be the time between a car accident and reporting a claim to the insurance company. Let T_2 be the time between the report of the claim and payment of the claim. The joint density function of T_1 and T_2, $f(t_1, t_2)$, is constant over the region $0 < t_1 < 6, 0 < t_2 < 6, t_1 + t_2 < 10$, and zero otherwise.

Calculate $E(T_1 + T_2)$, the expected time between a car accident and payment of the claim.

(A)   4.9
(B)   5.0
(C)   5.7
(D)   6.0
(E)   6.7

95. X and Y are independent random variables with common moment generating function $M(t) = \exp(t^2 / 2)$.

Let $W = X + Y$ and $Z = Y - X$.

Determine the joint moment generating function, $M(t_1, t_2)$ of W and Z.

(A) $\exp(2t_1^2 + 2t_2^2)$

(B) $\exp[(t_1 - t_2)^2]$

(C) $\exp[(t_1 + t_2)^2]$

(D) $\exp(2t_1 t_2)$

(E) $\exp(t_1^2 + t_2^2)$

96. A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists.

Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 (ticket cost + 50 penalty) to the tourist.

Calculate the expected revenue of the tour operator.

(A)   955
(B)   962
(C)   967
(D)   976
(E)   985

97. Let T_1 and T_2 represent the lifetimes in hours of two linked components in an electronic device. The joint density function for T_1 and T_2 is uniform over the region defined by $0 \le t_1 \le t_2 \le L$ where L is a positive constant.

Determine the expected value of the sum of the squares of T_1 and T_2.

(A) $\dfrac{L^2}{3}$

(B) $\dfrac{L^2}{2}$

(C) $\dfrac{2L^2}{3}$

(D) $\dfrac{3L^2}{4}$

(E) L^2

98. Let X_1, X_2, X_3 be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} 1/3, & x = 0 \\ 2/3, & x = 1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the moment generating function, $M(t)$, of $Y = X_1X_2X_3$.

(A) $\dfrac{19}{27} + \dfrac{8}{27}e^t$

(B) $1 + 2e^t$

(C) $\left(\dfrac{1}{3} + \dfrac{2}{3}e^t\right)^3$

(D) $\dfrac{1}{27} + \dfrac{8}{27}e^{3t}$

(E) $\dfrac{1}{3} + \dfrac{2}{3}e^{3t}$

99. An insurance policy pays a total medical benefit consisting of two parts for each claim. Let X represent the part of the benefit that is paid to the surgeon, and let Y represent the part that is paid to the hospital. The variance of X is 5000, the variance of Y is 10,000, and the variance of the total benefit, $X + Y$, is 17,000.

Due to increasing medical costs, the company that issues the policy decides to increase X by a flat amount of 100 per claim and to increase Y by 10% per claim.

Calculate the variance of the total benefit after these revisions have been made.

(A)   18,200
(B)   18,800
(C)   19,300
(D)   19,520
(E)   20,670

100. A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let X denote the number of luxury cars sold in a given day, and let Y denote the number of extended warranties sold.

$$P[X = 0, Y = 0] = 1/6$$
$$P[X = 1, Y = 0] = 1/12$$
$$P[X = 1, Y = 1] = 1/6$$
$$P[X = 2, Y = 0] = 1/12$$
$$P[X = 2, Y = 1] = 1/3$$
$$P[X = 2, Y = 2] = 1/6$$

Calculate the variance of X.

(A)   0.47
(B)   0.58
(C)   0.83
(D)   1.42
(E)   2.58

101. The profit for a new product is given by $Z = 3X - Y - 5$. X and Y are independent random variables with $\text{Var}(X) = 1$ and $\text{Var}(Y) = 2$.

Calculate $\text{Var}(Z)$.

(A)   1
(B)   5
(C)   7
(D)   11
(E)   16

102. A company has two electric generators. The time until failure for each generator follows an exponential distribution with mean 10. The company will begin using the second generator immediately after the first one fails.

Calculate the variance of the total time that the generators produce electricity.

(A)   10
(B)   20
(C)   50
(D)   100
(E)   200

103. In a small metropolitan area, annual losses due to storm, fire, and theft are assumed to be mutually independent, exponentially distributed random variables with respective means 1.0, 1.5, and 2.4.

Calculate the probability that the maximum of these losses exceeds 3.

(A)   0.002
(B)   0.050
(C)   0.159
(D)   0.287
(E)   0.414

104. A joint density function is given by

$$f(x, y) = \begin{cases} kx, & 0 < x < 1, \ 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

where k is a constant.

Calculate $\text{Cov}(X,Y)$.

(A)   −1/6
(B)   0

(C)   1/9
(D)   1/6
(E)   2/3

105. Let *X* and *Y* be continuous random variables with joint density function

$$f(x, y) = \begin{cases} \dfrac{8}{3}xy, & 0 \le x \le 1, \ x \le y \le 2x \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the covariance of *X* and *Y*.

(A)   0.04
(B)   0.25
(C)   0.67
(D)   0.80
(E)   1.24

106. Let *X* and *Y* denote the values of two stocks at the end of a five-year period. *X* is uniformly distributed on the interval (0, 12). Given *X* = *x*, *Y* is uniformly distributed on the interval (0, *x*).

Calculate Cov(*X*, *Y*) according to this model.

(A)   0
(B)   4
(C)   6
(D)   12
(E)   24

107. Let X denote the size of a surgical claim and let Y denote the size of the associated hospital claim. An actuary is using a model in which
$$E(X)=5,\ E(X^2)=27.4,\ E(Y)=7,\ E(Y^2)=51.4,\ Var(X+Y)=8.$$

Let $C_1 = X+Y$ denote the size of the combined claims before the application of a 20% surcharge on the hospital portion of the claim, and let C_2 denote the size of the combined claims after the application of that surcharge.

Calculate $Cov(C_1, C_2)$.

(A)   8.80
(B)   9.60
(C)   9.76
(D)   11.52
(E)   12.32

108. A device containing two key components fails when, and only when, both components fail. The lifetimes, T_1 and T_2 of these components are independent with common density function

$$f(t) = \begin{cases} e^{-t}, & 0 < t \\ 0, & \text{otherwise.} \end{cases}$$

The cost, X, of operating the device until failure is $2T_1 + T_2$. Let g be the density function for X.

Determine $g(x)$, for $x > 0$.

(A) $e^{-x/2} - e^{-x}$

(B) $2\left(e^{-x/2} - e^{-x}\right)$

(C) $\dfrac{x^2 e^{-x}}{2}$

(D) $\dfrac{e^{-x/2}}{2}$

(E) $\dfrac{e^{-x/3}}{3}$

109. A company offers earthquake insurance. Annual premiums are modeled by an exponential random variable with mean 2. Annual claims are modeled by an exponential random variable with mean 1. Premiums and claims are independent. Let X denote the ratio of claims to premiums, and let f be the density function of X.

Determine $f(x)$, where it is positive.

(A) $\dfrac{1}{2x+1}$

(B) $\dfrac{2}{(2x+1)^2}$

(C) e^{-x}

(D) $2e^{-2x}$

(E) xe^{-x}

110. Let X and Y be continuous random variables with joint density function

$$f(x,y)=\begin{cases}24xy, & 0<x<1,\,0<y<1-x\\0, & \text{otherwise.}\end{cases}$$

Calculate $P\left[Y<X\,\middle|\,X=\dfrac{1}{3}\right]$.

(A) 1/27
(B) 2/27
(C) 1/4
(D) 1/3
(E) 4/9

111. Once a fire is reported to a fire insurance company, the company makes an initial estimate, X, of the amount it will pay to the claimant for the fire loss. When the claim is finally settled, the company pays an amount, Y, to the claimant. The company has determined that X and Y have the joint density function

$$f(x,y) = \begin{cases} \dfrac{2}{x^2(x-1)} y^{-(2x-1)/(x-1)}, & x > 1, \ y > 1 \\ 0, & \text{otherwise.} \end{cases}$$

Given that the initial claim estimated by the company is 2, calculate the probability that the final settlement amount is between 1 and 3.

(A)   1/9
(B)   2/9
(C)   1/3
(D)   2/3
(E)   8/9

112. A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy.

Let X denote the proportion of employees who purchase the basic policy, and Y the proportion of employees who purchase the supplemental policy. Let X and Y have the joint density function $f(x,y) = 2(x + y)$ on the region where the density is positive.

Given that 10% of the employees buy the basic policy, calculate the probability that fewer than 5% buy the supplemental policy.

(A)   0.010
(B)   0.013
(C)   0.108
(D)   0.417
(E)   0.500

113. Two life insurance policies, each with a death benefit of 10,000 and a one-time premium of 500, are sold to a married couple, one for each person. The policies will expire at the end of the tenth year. The probability that only the wife will survive at least ten years is 0.025, the probability that only the husband will survive at least ten years is 0.01, and the probability that both of them will survive at least ten years is 0.96.

Calculate the expected excess of premiums over claims, given that the husband survives at least ten years.

(A)   350
(B)   385

(C)   397
(D)   870
(E)   897

114. A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let X denote the disease state (0 or 1) of a patient, and let Y denote the outcome of the diagnostic test. The joint probability function of X and Y is given by:

$$P[X = 0,\ Y = 0] = 0.800$$
$$P[X = 1,\ Y = 0] = 0.050$$
$$P[X = 0,\ Y = 1] = 0.025$$
$$P[X = 1,\ Y = 1] = 0.125$$

Calculate $\mathrm{Var}(Y\,|\,X = 1)$.

(A)   0.13
(B)   0.15
(C)   0.20
(D)   0.51
(E)   0.71

115. The stock prices of two companies at the end of any given year are modeled with random variables X and Y that follow a distribution with joint density function

$$f(x, y) = \begin{cases} 2x, & 0 < x < 1, \ x < y < x+1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the conditional variance of Y given that $X = x$.

(A) 1/12
(B) 7/6
(C) $x + 1/2$
(D) $x^2 - 1/6$
(E) $x^2 + x + 1/3$

116. An actuary determines that the annual number of tornadoes in counties P and Q are jointly distributed as follows:

		Annual number of tornadoes in county Q			
		0	1	2	3
Annual number	0	0.12	0.06	0.05	0.02
of tornadoes	1	0.13	0.15	0.12	0.03
in county P	2	0.05	0.15	0.10	0.02

Calculate the conditional variance of the annual number of tornadoes in county Q, given that there are no tornadoes in county P.

(A) 0.51
(B) 0.84
(C) 0.88
(D) 0.99
(E) 1.76

117. A company is reviewing tornado damage claims under a farm insurance policy. Let X be the portion of a claim representing damage to the house and let Y be the portion of the same claim representing damage to the rest of the property. The joint density function of X and Y is

$$f(x, y) = \begin{cases} 6[1-(x+y)], & x > 0, \ y > 0, \ x+y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the probability that the portion of a claim representing damage to the house is less than 0.2.

(A) 0.360

(B)   0.480
(C)   0.488
(D)   0.512
(E)   0.520

118. Let X and Y be continuous random variables with joint density function

$$f(x, y) = \begin{cases} 15y, & x^2 \le y \le x \\ 0, & \text{otherwise.} \end{cases}$$

Let g be the marginal density function of Y.

Determine which of the following represents g.

(A) $g(y) = \begin{cases} 15y, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

(B) $g(y) = \begin{cases} \dfrac{15y^2}{2}, & x^2 < y < x \\ 0, & \text{otherwise} \end{cases}$

(C) $g(y) = \begin{cases} \dfrac{15y^2}{2}, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

(D) $g(y) = \begin{cases} 15y^{3/2}(1 - y^{1/2}), & x^2 < y < x \\ 0, & \text{otherwise} \end{cases}$

(E) $g(y) = \begin{cases} 15y^{3/2}(1 - y^{1/2}), & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

119. An auto insurance policy will pay for damage to both the policyholder's car and the other driver's car in the event that the policyholder is responsible for an accident. The size of the payment for damage to the policyholder's car, X, has a marginal density function of 1 for $0 < x < 1$. Given $X = x$, the size of the payment for damage to the other driver's car, Y, has conditional density of 1 for $x < y < x + 1$.

Given that the policyholder is responsible for an accident, calculate the probability that the payment for damage to the other driver's car will be greater than 0.5.

(A)   3/8
(B)   1/2
(C)   3/4
(D)   7/8
(E)   15/16

120. An insurance policy is written to cover a loss X where X has density function

$$f(x) = \begin{cases} \dfrac{3}{8}x^2, & 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

The time (in hours) to process a claim of size x, where $0 \le x \le 2$, is uniformly distributed on the interval from x to $2x$.

Calculate the probability that a randomly chosen claim on this policy is processed in three hours or more.

(A)   0.17
(B)   0.25
(C)   0.32
(D)   0.58
(E)   0.83

121. Let X represent the age of an insured automobile involved in an accident. Let Y represent the length of time the owner has insured the automobile at the time of the accident.

X and Y have joint probability density function

$$f(x, y) = \begin{cases} \dfrac{1}{64}\left(10 - xy^2\right), & 2 \le x \le 10,\, 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected age of an insured automobile involved in an accident.

(A)   4.9
(B)   5.2
(C)   5.8
(D)   6.0
(E)   6.4

122. A device contains two circuits. The second circuit is a backup for the first, so the second is used only when the first has failed. The device fails when and only when the second circuit fails.

Let X and Y be the times at which the first and second circuits fail, respectively. X and Y have joint probability density function

$$f(x, y) = \begin{cases} 6e^{-x}e^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected time at which the device fails.

(A)   0.33
(B)   0.50
(C)   0.67
(D)   0.83
(E)   1.50

123. You are given the following information about N, the annual number of claims for a randomly selected insured:

$$P(N = 0) = \frac{1}{2}, \quad P(N = 1) = \frac{1}{3}, \quad P(N > 1) = \frac{1}{6}.$$

Let S denote the total annual claim amount for an insured. When $N = 1$, S is exponentially distributed with mean 5. When $N > 1$, S is exponentially distributed with mean 8.

Calculate $P(4 < S < 8)$.

(A)   0.04
(B)   0.08
(C)   0.12
(D)   0.24
(E)   0.25

124. The joint probability density for X and Y is

$$f(x, y) = \begin{cases} 2e^{-(x+2y)}, & x > 0, y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the variance of Y given that $X > 3$ and $Y > 3$.

(A)   0.25
(B)   0.50
(C)   1.00
(D)   3.25
(E)   3.50

125. The distribution of Y, given X, is uniform on the interval $[0, X]$. The marginal density of X is

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the conditional density of X, given $Y = y$ where positive.

(A)   1
(B)   2
(C) $2x$
(D) $1/y$
(E) $1/(1 - y)$

126. Under an insurance policy, a maximum of five claims may be filed per year by a policyholder. Let $p(n)$ be the probability that a policyholder files n claims during a given year, where $n = 0,1,2,3,4,5$. An actuary makes the following observations:

i) $p(n) \geq p(n+1)$ for $n = 0, 1, 2, 3, 4$.

ii) The difference between $p(n)$ and $p(n+1)$ is the same for $n = 0,1,2,3,4$.

iii) Exactly 40% of policyholders file fewer than two claims during a given year.

Calculate the probability that a random policyholder will file more than three claims during a given year.

(A)   0.14
(B)   0.16
(C)   0.27
(D)   0.29
(E)   0.33

127. The amounts of automobile losses reported to an insurance company are mutually independent, and each loss is uniformly distributed between 0 and 20,000. The company covers each such loss subject to a deductible of 5,000.

Calculate the probability that the total payout on 200 reported losses is between 1,000,000 and 1,200,000.

(A)   0.0803
(B)   0.1051
(C)   0.1799
(D)   0.8201
(E)   0.8575

128. An insurance agent offers his clients auto insurance, homeowners insurance and renters insurance. The purchase of homeowners insurance and the purchase of renters insurance are mutually exclusive. The profile of the agent's clients is as follows:

i)   17% of the clients have none of these three products.
ii)   64% of the clients have auto insurance.
iii)   Twice as many of the clients have homeowners insurance as have renters insurance.
iv)   35% of the clients have two of these three products.
v)   11% of the clients have homeowners insurance, but not auto insurance.

Calculate the percentage of the agent's clients that have both auto and renters insurance.

(A)   7%
(B)   10%
(C)   16%
(D)   25%
(E)   28%

129. The cumulative distribution function for health care costs experienced by a policyholder is modeled by the function

$$F(x) = \begin{cases} -e^{-\frac{x}{100}}, & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

The policy has a deductible of 20. An insurer reimburses the policyholder for 100% of health care costs between 20 and 120 less the deductible. Health care costs above 120 are reimbursed at 50%.

Let G be the cumulative distribution function of reimbursements given that the reimbursement is positive.
Calculate $G(115)$.

(A)   0.683
(B)   0.727
(C)   0.741
(D)   0.757
(E)   0.777

130. The value of a piece of factory equipment after three years of use is $100(0.5)^X$ where X is a random variable having moment generating function

$$M_X(t) = \frac{1}{1-2t}, \quad t < \frac{1}{2}.$$

Calculate the expected value of this piece of equipment after three years of use.

(A) 12.5
(B) 25.0
(C) 41.9
(D) 70.7
(E) 83.8

131. Let N_1 and N_2 represent the numbers of claims submitted to a life insurance company in April and May, respectively. The joint probability function of N_1 and N_2 is

$$p(n_1, n_2) = \begin{cases} \dfrac{3}{4}\left(\dfrac{1}{4}\right)^{n_1-1} e^{-n_1}\left(1-e^{-n_1}\right)^{n_2-1}, & n_1 = 1,2,3,..., \; n_2 = 1,2,3,... \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected number of claims that will be submitted to the company in May, given that exactly 2 claims were submitted in April.

(A) $\dfrac{3}{16}\left(e^2 - 1\right)$

(B) $\dfrac{3}{16}e^2$

(C) $\dfrac{3e}{4-e}$

(D) $e^2 - 1$

(E) e^2

132. A store has 80 modems in its inventory, 30 coming from Source A and the remainder from Source B. Of the modems from Source A, 20% are defective. Of the modems from Source B, 8% are defective.

Calculate the probability that exactly two out of a sample of five modems selected without replacement from the store's inventory are defective.

(A)   0.010
(B)   0.078
(C)   0.102
(D)   0.105
(E)   0.125

133. A man purchases a life insurance policy on his 40$^{\text{th}}$ birthday. The policy will pay 5000 if he dies before his 50$^{\text{th}}$ birthday and will pay 0 otherwise. The length of lifetime, in years from birth, of a male born the same year as the insured has the cumulative distribution function

$$F(t) = \begin{cases} 0, & t \le 0 \\ 1 - \exp\left(\dfrac{1-1.1^t}{1000}\right), & t > 0. \end{cases}$$

Calculate the expected payment under this policy.

(A)   333
(B)   348
(C)   421
(D)   549
(E)   574

134. A mattress store sells only king, queen and twin-size mattresses. Sales records at the store indicate that one-fourth as many queen-size mattresses are sold as king and twin-size mattresses combined. Records also indicate that three times as many king-size mattresses are sold as twin-size mattresses.

Calculate the probability that the next mattress sold is either king or queen-size.

(A)   0.12
(B)   0.15
(C)   0.80
(D)   0.85
(E)   0.95

135. The number of workplace injuries, *N*, occurring in a factory on any given day is Poisson distributed with mean λ. The parameter λ is a random variable that is determined by the level of activity in the factory, and is uniformly distributed on the interval [0, 3].

Calculate Var(*N*).

(A) λ
(B) 2λ
(C) 0.75
(D) 1.50
(E) 2.25

136. A fair die is rolled repeatedly. Let *X* be the number of rolls needed to obtain a 5 and *Y* the number of rolls needed to obtain a 6.

Calculate E(*X* | *Y* = 2).

(A) 5.0
(B) 5.2
(C) 6.0
(D) 6.6
(E) 6.8

137. Let *X* and *Y* be identically distributed independent random variables such that the moment generating function of *X* + *Y* is

$$M(t) = 0.09e^{-2t} + 0.24e^{-t} + 0.34 + 0.24e^{t} + 0.09e^{2t}, \ -\infty < t < \infty.$$

Calculate P[*X* ≤ 0].

(A) 0.33
(B) 0.34
(C) 0.50
(D) 0.67
(E) 0.70

138. A machine consists of two components, whose lifetimes have the joint density function

$$f(x, y) = \begin{cases} \dfrac{1}{50}, & x > 0, \ y > 0, \ x + y < 10 \\ 0, & \text{otherwise.} \end{cases}$$

The machine operates until both components fail.

Calculate the expected operational time of the machine.

(A) 1.7
(B) 2.5
(C) 3.3
(D) 5.0
(E) 6.7

139. A driver and a passenger are in a car accident. Each of them independently has probability 0.3 of being hospitalized. When a hospitalization occurs, the loss is uniformly distributed on [0, 1]. When two hospitalizations occur, the losses are independent.

Calculate the expected number of people in the car who are hospitalized, given that the total loss due to hospitalizations from the accident is less than 1.

(A) 0.510
(B) 0.534
(C) 0.600
(D) 0.628
(E) 0.800

140. Each time a hurricane arrives, a new home has a 0.4 probability of experiencing damage. The occurrences of damage in different hurricanes are mutually independent.

Calculate the mode of the number of hurricanes it takes for the home to experience damage from two hurricanes.

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

141. Thirty items are arranged in a 6-by-5 array as shown.

A_1	A_2	A_3	A_4	A_5
A_6	A_7	A_8	A_9	A_{10}
A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
A_{16}	A_{17}	A_{18}	A_{19}	A_{20}
A_{21}	A_{22}	A_{23}	A_{24}	A_{25}
A_{26}	A_{27}	A_{28}	A_{29}	A_{30}

Calculate the number of ways to form a set of three distinct items such that no two of the selected items are in the same row or same column.

(A)   200
(B)   760
(C)   1200
(D)   4560
(E)   7200

142. An auto insurance company is implementing a new bonus system. In each month, if a policyholder does not have an accident, he or she will receive a cash-back bonus of 5 from the insurer.

Among the 1,000 policyholders of the auto insurance company, 400 are classified as low-risk drivers and 600 are classified as high-risk drivers.

In each month, the probability of zero accidents for high-risk drivers is 0.80 and the probability of zero accidents for low-risk drivers is 0.90.

Calculate the expected bonus payment from the insurer to the 1000 policyholders in one year.

(A)   48,000
(B)   50,400
(C)   51,000
(D)   54,000
(E)   60,000

143. The probability that a member of a certain class of homeowners with liability and property coverage will file a liability claim is 0.04, and the probability that a member of this class will file a property claim is 0.10. The probability that a member of this class will file a liability claim but not a property claim is 0.01.

Calculate the probability that a randomly selected member of this class of homeowners will not file a claim of either type.

(A)   0.850
(B)   0.860
(C)   0.864
(D)   0.870
(E)   0.890

144. A client spends X minutes in an insurance agent's waiting room and Y minutes meeting with the agent. The joint density function of X and Y can be modeled by

$$f(x, y) = \begin{cases} \dfrac{1}{800} e^{-\frac{x}{40}} e^{-\frac{y}{20}}, & x > 0, \ y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Determine which of the following expressions represents the probability that a client spends less than 60 minutes at the agent's office.

(A) $\dfrac{1}{800} \int\limits_{0}^{40} \int\limits_{0}^{20} e^{-\frac{x}{40}} e^{-\frac{y}{20}} dy\, dx$

(B) $\dfrac{1}{800} \int\limits_{0}^{40} \int\limits_{0}^{20-x} e^{-\frac{x}{40}} e^{-\frac{y}{20}} dy\, dx$

(C) $\dfrac{1}{800} \int\limits_{0}^{20} \int\limits_{0}^{40-x} e^{-\frac{x}{40}} e^{-\frac{y}{20}} dy\, dx$

(D) $\dfrac{1}{800} \int\limits_{0}^{60} \int\limits_{0}^{60} e^{-\frac{x}{40}} e^{-\frac{y}{20}} dy\, dx$

(E) $\dfrac{1}{800} \int\limits_{0}^{60} \int\limits_{0}^{60-x} e^{-\frac{x}{40}} e^{-\frac{y}{20}} dy\, dx$

145. New dental and medical plan options will be offered to state employees next year. An actuary uses the following density function to model the joint distribution of the proportion X of state employees who will choose Dental Option 1 and the proportion Y who will choose Medical Option 1 under the new plan options:

$$f(x, y) = \begin{cases} 0.50, & 0 < x < 0.5, 0 < y < 0.5 \\ 1.25, & 0 < x < 0.5, 0.5 < y < 1 \\ 1.50, & 0.5 < x < 1, 0 < y < 0.5 \\ 0.75, & 0.5 < x < 1, 0.5 < y < 1. \end{cases}$$

Calculate Var $(Y \mid X = 0.75)$.

(A)   0.000
(B)   0.061
(C)   0.076
(D)   0.083
(E)   0.141

146. A survey of 100 TV viewers revealed that over the last year:

i)   34 watched CBS.
ii)   15 watched NBC.
iii)   10 watched ABC.
iv)   7 watched CBS and NBC.
v)   6 watched CBS and ABC.
vi)   5 watched NBC and ABC.
vii)   4 watched CBS, NBC, and ABC.
viii)   18 watched HGTV, and of these, none watched CBS, NBC, or ABC.

Calculate how many of the 100 TV viewers did not watch any of the four channels (CBS, NBC, ABC or HGTV).

(A)   1
(B)   37
(C)   45
(D)   55
(E)   82

147. The amount of a claim that a car insurance company pays out follows an exponential distribution. By imposing a deductible of d, the insurance company reduces the expected claim payment by 10%.

Calculate the percentage reduction on the variance of the claim payment.

(A)   1%
(B)   5%
(C)   10%
(D)   20%
(E)   25%

148. The number of hurricanes that will hit a certain house in the next ten years is Poisson distributed with mean 4.

Each hurricane results in a loss that is exponentially distributed with mean 1000. Losses are mutually independent and independent of the number of hurricanes.

Calculate the variance of the total loss due to hurricanes hitting this house in the next ten years.

(A)   4,000,000
(B)   4,004,000
(C)   8,000,000
(D)   16,000,000
(E)   20,000,000

149. A motorist makes three driving errors, each independently resulting in an accident with probability 0.25.

Each accident results in a loss that is exponentially distributed with mean 0.80. Losses are mutually independent and independent of the number of accidents.

The motorist's insurer reimburses 70% of each loss due to an accident.

Calculate the variance of the total unreimbursed loss the motorist experiences due to accidents resulting from these driving errors.

(A)   0.0432
(B)   0.0756
(C)   0.1782
(D)   0.2520
(E)   0.4116

150. An automobile insurance company issues a one-year policy with a deductible of 500. The probability is 0.8 that the insured automobile has no accident and 0.0 that the automobile has more than one accident. If there is an accident, the loss before application of the deductible is exponentially distributed with mean 3000.

Calculate the 95th percentile of the insurance company payout on this policy.

(A)   3466
(B)   3659
(C)   4159
(D)   8487
(E)   8987

151. From 27 pieces of luggage, an airline luggage handler damages a random sample of four.

The probability that exactly one of the damaged pieces of luggage is insured is twice the probability that none of the damaged pieces are insured.

Calculate the probability that exactly two of the four damaged pieces are insured.

(A)   0.06
(B)   0.13
(C)   0.27
(D)   0.30
(E)   0.31

152. Automobile policies are separated into two groups: low-risk and high-risk. Actuary Rahul examines low-risk policies, continuing until a policy with a claim is found and then stopping. Actuary Toby follows the same procedure with high-risk policies. Each low-risk policy has a 10% probability of having a claim. Each high-risk policy has a 20% probability of having a claim. The claim statuses of polices are mutually independent.

Calculate the probability that Actuary Rahul examines fewer policies than Actuary Toby.

(A)   0.2857
(B)   0.3214
(C)   0.3333
(D)   0.3571
(E)   0.4000

153. Let X represent the number of customers arriving during the morning hours and let Y represent the number of customers arriving during the afternoon hours at a diner.
You are given:
i) X and Y are Poisson distributed.
ii)   The first moment of X is less than the first moment of Y by 8.
iii)   The second moment of X is 60% of the second moment of Y.

Calculate the variance of Y.

(A)   4
(B)   12
(C)   16
(D)   27
(E)   35

154. In a certain game of chance, a square board with area 1 is colored with sectors of either red or blue. A player, who cannot see the board, must specify a point on the board by giving an x-coordinate and a y-coordinate. The player wins the game if the specified point is in a blue sector. The game can be arranged with any number of red sectors, and the red sectors are designed so that

$$R_i \;=\; \left(\frac{9}{20}\right)^i, \text{ where } R_i \text{ is the area of the } i^{th} \text{ red sector.}$$

Calculate the minimum number of red sectors that makes the chance of a player winning less than 20%.

(A)   3
(B)   4
(C)   5
(D)   6
(E)   7

155. Automobile claim amounts are modeled by a uniform distribution on the interval [0, 10,000]. Actuary A reports X, the claim amount divided by 1000. Actuary B reports Y, which is X rounded to the nearest integer from 0 to 10.

Calculate the absolute value of the difference between the 4th moment of X and the 4th moment of Y.

(A)   0
(B)   33
(C)   296
(D)   303
(E)   533

156. The probability of x losses occurring in year 1 is $(0.5)^{x+1}$ for $x = 0,1,2,\ldots$.

The probability of y losses in year 2 given x losses in year 1 is given by the table:

Number of losses in year 1 (x)	Number of losses in year 2 (y) given x losses in year 1				
	0	1	2	3	4+
0	0.60	0.25	0.05	0.05	0.05
1	0.45	0.30	0.10	0.10	0.05
2	0.25	0.30	0.20	0.20	0.05
3	0.15	0.20	0.20	0.30	0.15
4+	0.05	0.15	0.25	0.35	0.20

Calculate the probability of exactly 2 losses in 2 years.
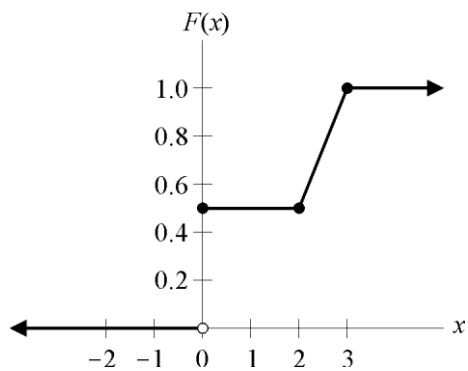
(A)   0.025
(B)   0.031
(C)   0.075
(D)   0.100
(E)   0.131

157. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \dfrac{p-1}{x^p}, & x > 1 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the value of p such that $E(X) = 2$.

(A)   1
(B)   2.5
(C)   3
(D)   5
(E)   There is no such p.

158. The figure below shows the cumulative distribution function of a random variable, X.

Calculate $E(X)$.

(A)   0.00
(B)   0.50
(C)   1.00
(D)   1.25
(E)   2.50

159. Two fair dice are rolled. Let X be the absolute value of the difference between the two numbers on the dice.

Calculate the probability that $X < 3$.

(A) 2/9
(B) 1/3
(C) 4/9
(D) 5/9
(E) 2/3

160. An actuary analyzes a company's annual personal auto claims, M, and annual commercial auto claims, N. The analysis reveals that $\text{Var}(M) = 1600$, $\text{Var}(N) = 900$, and the correlation between M and N is 0.64.

Calculate $\text{Var}(M + N)$.

(A) 768
(B) 2500
(C) 3268
(D) 4036
(E) 4420

161. An auto insurance policy has a deductible of 1 and a maximum claim payment of 5. Auto loss amounts follow an exponential distribution with mean 2.

Calculate the expected claim payment made for an auto loss.

(A) $0.5e^{-2} - 0.5e^{-12}$

(B) $2e^{-\frac{1}{2}} - 7e^{-3}$

(C) $2e^{-\frac{1}{2}} - 2e^{-3}$

(D) $2e^{-\frac{1}{2}}$

(E) $3e^{-\frac{1}{2}} - 2e^{-3}$

162. The joint probability density function of X and Y is given by

$$f(x, y) = \begin{cases} \dfrac{x+y}{8}, & 0 < x < 2, 0 < y < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the variance of $(X + Y)/2$.

(A)　10/72
(B)　11/72
(C)　12/72
(D)　20/72
(E)　22/72

163. A student takes a multiple-choice test with 40 questions. The probability that the student answers a given question correctly is 0.5, independent of all other questions. The probability that the student answers more than N questions correctly is greater than 0.10. The probability that the student answers more than $N + 1$ questions correctly is less than 0.10.

Calculate N using a normal approximation with the continuity correction.

(A)　23
(B)　25
(C)　32
(D)　33
(E)　35

164. In each of the months June, July, and August, the number of accidents occurring in that month is modeled by a Poisson random variable with mean 1. In each of the other 9 months of the year, the number of accidents occurring is modeled by a Poisson random variable with mean 0.5. Assume that these 12 random variables are mutually independent.

Calculate the probability that exactly two accidents occur in July through November.

(A)　0.084
(B)　0.185
(C)　0.251
(D)　0.257
(E)　0.271

165. Two claimants place calls simultaneously to an insurer's claims call center. The times X and Y, in minutes, that elapse before the respective claimants get to speak with call center representatives are independently and identically distributed. The moment generating function of each random variable is

$$M(t) = \left(\frac{1}{1-1.5t}\right)^2, \quad t < \frac{2}{3}.$$

Calculate the standard deviation of $X + Y$.

(A)   2.1
(B)   3.0
(C)   4.5
(D)   6.7
(E)   9.0

166. An airport purchases an insurance policy to offset costs associated with excessive amounts of snowfall. For every full ten inches of snow in excess of 40 inches during the winter season, the insurer pays the airport 300 up to a policy maximum of 700.

The following table shows the probability function for the random variable X of annual (winter season) snowfall, in inches, at the airport.

Inches	[0,20)	[20,30)	[30,40)	[40,50)	[50,60)	[60,70)	[70,80)	[80,90)	[90,inf)
Probability	0.06	0.18	0.26	0.22	0.14	0.06	0.04	0.04	0.00

Calculate the standard deviation of the amount paid under the policy.

(A)   134
(B)   235
(C)   271
(D)   313
(E)   352

167. Damages to a car in a crash are modeled by a random variable with density function

$$f(x) = \begin{cases} c(x^2 - 60x + 800), & 0 < x < 20 \\ 0, & \text{otherwise} \end{cases}$$

where c is a constant.

A particular car is insured with a deductible of 2. This car was involved in a crash with resulting damages in excess of the deductible.

Calculate the probability that the damages exceeded 10.

(A)   0.12
(B)   0.16
(C)   0.20
(D)   0.26
(E)   0.78

168. Two fair dice, one red and one blue, are rolled.

Let A be the event that the number rolled on the red die is odd.
Let B be the event that the number rolled on the blue die is odd.
Let C be the event that the sum of the numbers rolled on the two dice is odd.

Determine which of the following is true.

(A)   A, B, and C are not mutually independent, but each pair is independent.
(B)   A, B, and C are mutually independent.
(C)   Exactly one pair of the three events is independent.
(D)   Exactly two of the three pairs are independent.
(E)   No pair of the three events is independent.

169. An urn contains four fair dice. Two have faces numbered 1, 2, 3, 4, 5, and 6; one has faces numbered 2, 2, 4, 4, 6, and 6; and one has all six faces numbered 6. One of the dice is randomly selected from the urn and rolled. The same die is rolled a second time.

Calculate the probability that a 6 is rolled both times.

(A)   0.174
(B)   0.250
(C)   0.292
(D)   0.380
(E)   0.417

170. An insurance agent meets twelve potential customers independently, each of whom is equally likely to purchase an insurance product. Six are interested only in auto insurance, four are interested only in homeowners insurance, and two are interested only in life insurance.

The agent makes six sales.

Calculate the probability that two are for auto insurance, two are for homeowners insurance, and two are for life insurance.

(A)   0.001
(B)   0.024
(C)   0.069
(D)   0.097
(E)   0.500

171. The return on two investments, X and Y, follows the joint probability density function

$$f(x, y) = \begin{cases} 1/2, & 0 < |x| + |y| < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate Var (X).

(A)   1/6
(B)   1/3
(C)   1/2
(D)   2/3
(E)   5/6

172. A policyholder has probability 0.7 of having no claims, 0.2 of having exactly one claim, and 0.1 of having exactly two claims. Claim amounts are uniformly distributed on the interval [0, 60] and are independent. The insurer covers 100% of each claim.

Calculate the probability that the total benefit paid to the policyholder is 48 or less.

(A) 0.320
(B) 0.400
(C) 0.800
(D) 0.892
(E) 0.924

173. In a given region, the number of tornadoes in a one-week period is modeled by a Poisson distribution with mean 2. The numbers of tornadoes in different weeks are mutually independent.

Calculate the probability that fewer than four tornadoes occur in a three-week period.

(A) 0.13
(B) 0.15
(C) 0.29
(D) 0.43
(E) 0.86

174. An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail.

Calculate the probability that the system will overheat.

(A) 0.007
(B) 0.045
(C) 0.098
(D) 0.135
(E) 0.143

175. An insurance company's annual profit is normally distributed with mean 100 and variance 400.

Let Z be normally distributed with mean 0 and variance 1 and let F be the cumulative distribution function of Z.

Determine the probability that the company's profit in a year is at most 60, given that the profit in the year is positive.

(A) $1 - F(2)$

(B) $F(2)/F(5)$

(C) $[1 - F(2)]/F(5)$

(D) $[F(0.25) - F(0.1)]/F(0.25)$

(E) $[F(5) - F(2)]/F(5)$

176. In a group of health insurance policyholders, 20% have high blood pressure and 30% have high cholesterol. Of the policyholders with high blood pressure, 25% have high cholesterol.

A policyholder is randomly selected from the group.

Calculate the probability that a policyholder has high blood pressure, given that the policyholder has high cholesterol.

(A)   1/6
(B)   1/5
(C)   1/4
(D)   2/3
(E)   5/6

177. In a group of 25 factory workers, 20 are low-risk and five are high-risk.

Two of the 25 factory workers are randomly selected without replacement.

Calculate the probability that exactly one of the two selected factory workers is low-risk.

(A)   0.160
(B)   0.167
(C)   0.320
(D)   0.333
(E)   0.633

178. The proportion X of yearly dental claims that exceed 200 is a random variable with probability density function

$$f(x) = \begin{cases} 60x^3(1-x)^2, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $\text{Var}[X/(1 - X)]$

(A)   149/900
(B)   10/7
(C)   6
(D)   8
(E)   10

179. This year, a medical insurance policyholder has probability 0.70 of having no emergency room visits, 0.85 of having no hospital stays, and 0.61 of having neither emergency room visits nor hospital stays

Calculate the probability that the policyholder has at least one emergency room visit and at least one hospital stay this year.

(A)   0.045
(B)   0.060
(C)   0.390
(D)   0.667
(E)   0.840

180. An insurer offers a travelers insurance policy. Losses under the policy are uniformly distributed on the interval [0, 5].

The insurer reimburses a policyholder for a loss up to a maximum of 4.

Determine the cumulative distribution function, F, of the benefit that the insurer pays a policyholder who experiences exactly one loss under the policy.

(A) $F(x) = \begin{cases} 0, & x < 0 \\ 0.20x, & 0 \le x < 4 \\ 1 & x \ge 4 \end{cases}$

(B) $F(x) = \begin{cases} 0, & x < 0 \\ 0.20x, & 0 \le x < 5 \\ 1 & x \ge 5 \end{cases}$

(C) $F(x) = \begin{cases} 0, & x < 0 \\ 0.25x, & 0 \le x < 4 \\ 1 & x \ge 4 \end{cases}$

(D) $F(x) = \begin{cases} 0, & x < 0 \\ 0.25x, & 0 \le x < 5 \\ 1 & x \ge 5 \end{cases}$

(E) $F(x) = \begin{cases} 0, & x < 1 \\ 0.25x, & 1 \le x < 5 \\ 1 & x \ge 5 \end{cases}$

181. A company issues auto insurance policies. There are 900 insured individuals. Fifty-four percent of them are male. If a female is randomly selected from the 900, the probability she is over 25 years old is 0.43. There are 395 total insured individuals over 25 years old.

A person under 25 years old is randomly selected.

Calculate the probability that the person selected is male.

(A) 0.47
(B) 0.53
(C) 0.54
(D) 0.55
(E) 0.56

182. An insurance company insures red and green cars. An actuary compiles the following data:

Color of car	Red	Green
Number insured	300	700
Probability an accident occurs	0.10	0.05
Probability that the claim exceeds the deductible, given an accident occurs from this group	0.90	0.80

The actuary randomly picks a claim from all claims that exceed the deductible.

Calculate the probability that the claim is on a red car.

(A) 0.300
(B) 0.462
(C) 0.491
(D) 0.667
(E) 0.692

183. X is a random variable with probability density function

$$f(x) = \begin{cases} e^{-2x}, & x \geq 0 \\ 2e^{4x}, & x < 0 \end{cases}$$

Let $T = X^2$.

Determine the probability density function for T for positive values of t.

(A) $f(t) = \dfrac{e^{-2\sqrt{t}}}{2\sqrt{t}} + \dfrac{e^{-4\sqrt{t}}}{\sqrt{t}}$

(B) $f(t) = \dfrac{e^{-2\sqrt{t}}}{2\sqrt{t}}$

(C) $f(t) = e^{-2t} + 2e^{-4t}$

(D) $f(t) = 2te^{-2t^2} + 4te^{-4t^2}$

(E) $f(t) = 2te^{-2t^2}$

184. George and Paul play a betting game. Each chooses an integer from 1 to 20 (inclusive) at random. If the two numbers differ by more than 3, George wins the bet. Otherwise, Paul wins the bet.

Calculate the probability that Paul wins the bet.

(A)   0.27
(B)   0.32
(C)   0.40
(D)   0.48
(E)   0.66

185. A student takes an examination consisting of 20 true-false questions. The student knows the answer to N of the questions, which are answered correctly, and guesses the answers to the rest. The conditional probability that the student knows the answer to a question, given that the student answered it correctly, is 0.824..

Calculate N.

(A)   8
(B)   10
(C)   14
(D)   16
(E)   18

186. The minimum force required to break a particular type of cable is normally distributed with mean 12,432 and standard deviation 25. A random sample of 400 cables of this type is selected.

Calculate the probability that at least 349 of the selected cables will not break under a force of 12,400.

(A)   0.62
(B)   0.67
(C)   0.92
(D)   0.97
(E)   1.00

187. The number of policies that an agent sells has a Poisson distribution with modes at 2 and 3.

K is the smallest number such that the probability of selling more than *K* policies is less than 25%.

Calculate *K*.

(A)   1
(B)   2
(C)   3
(D)   4
(E)   5

188. Two fair dice are tossed. One die is red and one die is green.

Calculate the probability that the sum of the numbers on the two dice is an odd number given that the number that shows on the red die is larger than the number that shows on the green die.

(A)   1/4
(B)   5/12
(C)   3/7
(D)   1/2
(E)   3/5

189. In 1982 Abby's mother scored at the 93rd percentile in the math SAT exam. In 1982 the mean score was 503 and the variance of the scores was 9604.

In 2008 Abby took the math SAT and got the same numerical score as her mother had received 26 years before. In 2008 the mean score was 521 and the variance of the scores was 10,201.

Math SAT scores are normally distributed and stated in multiples of ten.

Calculate the percentile for Abby's score.

(A)   89th
(B)   90th
(C)   91st
(D)   92nd
(E)   93rd

190. A certain brand of refrigerator has a useful life that is normally distributed with mean 10 years and standard deviation 3 years. The useful lives of these refrigerators are independent.

Calculate the probability that the total useful life of two randomly selected refrigerators will exceed 1.9 times the useful life of a third randomly selected refrigerator.

(A) 0.407
(B) 0.444
(C) 0.556
(D) 0.593
(E) 0.604

191. A fire in an apartment building results in a loss, X, to the owner and a loss, Y, to the tenants. The variables X and Y have a bivariate normal distribution with $E(X) = 40$, $Var(X) = 76$, $E(Y) = 30$, $Var(Y) = 32$, and $Var(X \mid Y = 28.5) = 57$.

Calculate $Var(Y \mid X = 25)$.

(A) 13
(B) 24
(C) 32
(D) 50
(E) 57

192. Losses covered by a flood insurance policy are uniformly distributed on the interval [0, 2]. The insurer pays the amount of the loss in excess of a deductible d.

The probability that the insurer pays at least 1.20 on a random loss is 0.30.

Calculate the probability that the insurer pays at least 1.44 on a random loss.

(A) 0.06
(B) 0.16
(C) 0.18
(D) 0.20
(E) 0.28

193. The lifespan, in years, of a certain computer is exponentially distributed. The probability that its lifespan exceeds four years is 0.30.

Let $f(x)$ represent the density function of the computer's lifespan, in years, for $x > 0$.

Determine which of the following is an expression for $f(x)$.

(A)　　$1-(0.3)^{-x/4}$

(B)　　$1-(0.7)^{x/4}$

(C)　　$1-(0.3)^{x/4}$

(D)　　$-\dfrac{\ln 0.7}{4}(0.7)^{x/4}$

(E)　　$-\dfrac{\ln 0.3}{4}(0.3)^{x/4}$

194. The lifetime of a light bulb has density function, f, where $f(x)$ is proportional to

$$\frac{x^2}{1+x^3}, \quad 0 < x < 5, \text{ and } 0, \text{ otherwise.}$$

Calculate the mode of this distribution.

(A)　　0.00
(B)　　0.79
(C)　　1.26
(D)　　4.42
(E)　　5.00

195. An insurer's medical reimbursements have density function f, where $f(x)$ is proportional to

xe^{-x^2}, for $0 < x < 1$, and 0, otherwise.

Calculate the mode of the medical reimbursements.

(A)　　0.00
(B)　　0.50
(C)　　0.71
(D)　　0.84
(E)　　1.00

196. A company has five employees on its health insurance plan. Each year, each employee independently has an 80% probability of no hospital admissions. If an employee requires one or more hospital admissions, the number of admissions is modeled by a geometric distribution with a mean of 1.50. The numbers of hospital admissions of different employees are mutually independent.

Each hospital admission costs 20,000.

Calculate the probability that the company's total hospital costs in a year are less than 50,000.

(A)   0.41
(B)   0.46
(C)   0.58
(D)   0.69
(E)   0.78

197. On any given day, a certain machine has either no malfunctions or exactly one malfunction. The probability of malfunction on any given day is 0.40. Machine malfunctions on different days are mutually independent.

Calculate the probability that the machine has its third malfunction on the fifth day, given that the machine has not had three malfunctions in the first three days.

(A)   0.064
(B)   0.138
(C)   0.148
(D   0.230
(E)   0.246

198. In a certain group of cancer patients, each patient's cancer is classified in exactly one of the following five stages: stage 0, stage 1, stage 2, stage 3, or stage 4.

i)   75% of the patients in the group have stage 2 or lower.
ii) 80% of the patients in the group have stage 1 or higher.
iii) 80% of the patients in the group have stage 0, 1, 3, or 4.

One patient from the group is randomly selected.

Calculate the probability that the selected patient's cancer is stage 1.

(A)   0.20
(B)   0.25
(C)   0.35
(D)   0.48
(E)   0.65

199. A car is new at the beginning of a calendar year. The time, in years, before the car experiences its first failure is exponentially distributed with mean 2.

Calculate the probability that the car experiences its first failure in the last quarter of some calendar year.

(A)   0.081
(B)   0.088
(C)   0.102
(D)   0.205
(E)   0.250

200. In a shipment of 20 packages, 7 packages are damaged. The packages are randomly inspected, one at a time, without replacement, until the fourth damaged package is discovered.

Calculate the probability that exactly 12 packages are inspected.

(A)   0.079
(B)   0.119
(C)   0.237
(D)   0.243
(E)   0.358

201. A theme park conducts a study of families that visit the park during a year. The fraction of such families of size m is $\dfrac{8-m}{28}$, $m = 1, 2, 3, 4, 5, 6$, and 7.

For a family of size m that visits the park, the number of members of the family that ride the roller coaster follows a discrete uniform distribution on the set $\{1, \dots, m\}$.

Calculate the probability that a family visiting the park has exactly six members, given that exactly five members of the family ride the roller coaster.

(A) 0.17
(B) 0.21
(C) 0.24
(D) 0.28
(E) 0.31

202. The following information is given about a group of high-risk borrowers.

i) Of all these borrowers, 30% defaulted on at least one student loan.
ii) Of the borrowers who defaulted on at least one car loan, 40% defaulted on at least one student loan.
iii) Of the borrowers who did not default on any student loans, 28% defaulted on at least one car loan.

A statistician randomly selects a borrower from this group and observes that the selected borrower defaulted on at least one student loan.

Calculate the probability that the selected borrower defaulted on at least one car loan.

(A) 0.33
(B) 0.40
(C) 0.44
(D) 0.65
(E) 0.72

203. A machine has two components and fails when both components fail. The number of years from now until the first component fails, X, and the number of years from now until the machine fails, Y, are random variables with joint density function

$$f(x, y) = \begin{cases} \dfrac{1}{18} e^{-(x+y)/6}, & 0 < x < y \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $\operatorname{Var}(Y \mid X = 2)$.

(A) 6
(B) 8
(C) 36
(D) 45
(E) 64

204. The elapsed time, T, between the occurrence and the reporting of an accident has probability density function

$$f(t) = \begin{cases} \dfrac{8t - t^2}{72}, & 0 < t < 6 \\ 0, & \text{otherwise.} \end{cases}$$

Given that $T = t$, the elapsed time between the reporting of the accident and payment by the insurer is uniformly distributed on $[2 + t, 10]$.

Calculate the probability that the elapsed time between the occurrence of the accident and payment by the insurer is less than 4.

(A) 0.005
(B) 0.023
(C) 0.033
(D) 0.035
(E) 0.133

205. The time until failure, T, of a product is modeled by a uniform distribution on $[0, 10]$. An extended warranty pays a benefit of 100 if failure occurs between time $t = 1.5$ and $t = 8$. The present value, W, of this benefit is

$$W = \begin{cases} 0, & \text{for } 0 \leq T < 1.5 \\ 100e^{-0.04T}, & \text{for } 1.5 \leq T < 8 \\ 0, & \text{for } 8 \leq T \leq 10. \end{cases}$$

Calculate $P(W < 79)$.

(A)   0.21
(B)   0.41
(C)   0.44
(D)   0.56
(E)   0.59

206. An insurance company issues policies covering damage to automobiles. The amount of damage is modeled by a uniform distribution on $[0, b]$.

The policy payout is subject to a deductible of $b/10$.

A policyholder experiences automobile damage.

Calculate the ratio of the standard deviation of the policy payout to the standard deviation of the amount of the damage.

(A)   0.8100
(B)   0.9000
(C)   0.9477
(D)   0.9487
(E)   0.9735

207   A policyholder purchases automobile insurance for two years. Define the following events:

F = the policyholder has exactly one accident in year one.
G = the policyholder has one or more accidents in year two.

Define the following events:

i)   The policyholder has exactly one accident in year one and has more than one accident in year two.
ii) The policyholder has at least two accidents during the two-year period.
iii) The policyholder has exactly one accident in year one and has at least one accident in year two.
iv) The policyholder has exactly one accident in year one and has a total of two or more accidents in the two-year period.
v)   The policyholder has exactly one accident in year one and has more accidents in year two than in year one.

Determine the number of events from the above list of five that are the same as $F \cap G$.

(A)   None
(B)   Exactly one
(C)   Exactly two
(D)   Exactly three
(E)   All

208.   An insurance company categorizes its policyholders into three mutually exclusive groups: high-risk, medium-risk, and low-risk. An internal study of the company showed that 45% of the policyholders are low-risk and 35% are medium-risk. The probability of death over the next year, given that a policyholder is high-risk is two times the probability of death of a medium-risk policyholder. The probability of death over the next year, given that a policyholder is medium-risk is three times the probability of death of a low-risk policyholder. The probability of death of a randomly selected policyholder, over the next year, is 0.009.

Calculate the probability of death of a policyholder over the next year, given that the policyholder is high-risk.

(A)   0.0025
(B)   0.0200
(C)   0.1215
(D)   0.2000
(E)   0.3750

209. A policy covers a gas furnace for one year. During that year, only one of three problems can occur:

i) The igniter switch may need to be replaced at a cost of 60. There is a 0.10 probability of this.

ii) The pilot light may need to be replaced at a cost of 200. There is a 0.05 probability of this.

iii) The furnace may need to be replaced at a cost of 3000. There is a 0.01 probability of this.

Calculate the deductible that would produce an expected claim payment of 30.

(A) 100
(B) At least 100 but less than 150
(C) At least 150 but less than 200
(D) At least 200 but less than 250
(E) At least 250

210. On a block of ten houses, k are not insured. A tornado randomly damages three houses on the block.

The probability that none of the damaged houses are insured is 1/120.

Calculate the probability that at most one of the damaged houses is insured.

(A) 1/5
(B) 7/40
(C) 11/60
(D) 49/60
(E) 119/120

211. In a casino game, a gambler selects four different numbers from the first twelve positive integers. The casino then randomly draws nine numbers without replacement from the first twelve positive integers. The gambler wins the jackpot if the casino draws all four of the gambler's selected numbers.

Calculate the probability that the gambler wins the jackpot.

(A) 0.002
(B) 0.255
(C) 0.296
(D) 0.573
(E) 0.625

212. The number of days an employee is sick each month is modeled by a Poisson distribution with mean 1. The numbers of sick days in different months are mutually independent.

Calculate the probability that an employee is sick more than two days in a three-month period.

(A)   0.199
(B)   0.224
(C)   0.423
(D)   0.577
(E)   0.801

213. The number of traffic accidents per week at intersection Q has a Poisson distribution with mean 3. The number of traffic accidents per week at intersection R has a Poisson distribution with mean 1.5.

Let *A* be the probability that the number of accidents at intersection Q exceeds its mean. Let *B* be the corresponding probability for intersection R.

Calculate *B* – *A*.

(A)   0.00
(B)   0.09
(C)   0.13
(D)   0.19
(E)   0.31

214. Losses due to accidents at an amusement park are exponentially distributed. An insurance company offers the park owner two different policies, with different premiums, to insure against losses due to accidents at the park.

Policy A has a deductible of 1.44. For a random loss, the probability is 0.640 that under this policy, the insurer will pay some money to the park owner. Policy B has a deductible of *d*. For a random loss, the probability is 0.512 that under this policy, the insurer will pay some money to the park owner.

Calculate *d*.

(A)   0.960
(B)   1.152
(C)   1.728
(D)   1.800
(E)   2.160

215. The distribution of the size of claims paid under an insurance policy has probability density function

$$f(x) = \begin{cases} cx^a, & 0 < x < 5 \\ 0, & \text{otherwise,} \end{cases}$$

Where $a > 0$ and $c > 0$.

For a randomly selected claim, the probability that the size of the claim is less than 3.75 is 0.4871.

Calculate the probability that the size of a randomly selected claim is greater than 4.

(A)   0.404
(B)   0.428
(C)   0.500
(D)   0.572
(E)   0.596

216. Company XYZ provides a warranty on a product that it produces. Each year, the number of warranty claims follows a Poisson distribution with mean c. The probability that no warranty claims are received in any given year is 0.60.

Company XYZ purchases an insurance policy that will reduce its overall warranty claim payment costs. The insurance policy will pay nothing for the first warranty claim received and 5000 for each claim thereafter until the end of the year.

Calculate the expected amount of annual insurance policy payments to Company XYZ.

(A)   554
(B)   872
(C)   1022
(D)   1354
(E)   1612

217. For a certain health insurance policy, losses are uniformly distributed on the interval $[0, b]$. The policy has a deductible of 180 and the expected value of the unreimbursed portion of a loss is 144.

Calculate b.

(A) 236
(B) 288
(C) 388
(D) 450
(E) 468

218. The working lifetime, in years, of a particular model of bread maker is normally distributed with mean 10 and variance 4.

Calculate the 12th percentile of the working lifetime, in years.

(A) 5.30
(B) 7.65
(C) 8.41
(D) 12.35
(E) 14.70

219. The profits of life insurance companies A and B are normally distributed with the same mean. The variance of company B's profit is 2.25 times the variance of company A's profit. The 14th percentile of company A's profit is the same as the pth percentile of company B's profit.

Calculate p.

(A) 5.3
(B) 9.3
(C) 21.0
(D) 23.6
(E) 31.6

220. The distribution of values of the retirement package offered by a company to new employees is modeled by the probability density function

$$f(x) = \begin{cases} \dfrac{1}{5}\, e^{-\frac{1}{5}(x-5)}, & x > 5 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the variance of the retirement package value for a new employee, given that the value is at least 10.

(A)   15
(B)   20
(C)   25
(D)   30
(E)   35

221. Insurance companies A and B each earn an annual profit that is normally distributed with the same positive mean. The standard deviation of company A's annual profit is one half of its mean.

In a given year, the probability that company B has a loss (negative profit) is 0.9 times the probability that company A has a loss.

Calculate the ratio of the standard deviation of company B's annual profit to the standard deviation of company A's annual profit.

(A)   0.49
(B)   0.90
(C)   0.98
(D)   1.11
(E)   1.71

222. Let *X* represent the number of policies sold by an agent in a day. The moment generating function of *X* is

$$M(t) = 0.45e^t + 0.35e^{2t} + 0.15e^{3t} + 0.05e^{4t}, \quad \text{for} -\infty < t < \infty.$$

Calculate the standard deviation of *X*.

(A)   0.76
(B)   0.87
(C)   1.48
(D)   1.80
(E)   4.00

223. Claim amounts at an insurance company are independent of one another. In year one, claim amounts are modeled by a normal random variable *X* with mean 100 and standard deviation 25. In year two, claim amounts are modeled by the random variable $Y = 1.04X + 5$.

Calculate the probability that a random sample of 25 claim amounts in year two average between 100 and 110.

(A)   0.48
(B)   0.53
(C)   0.54
(D)   0.67
(E)   0.68

224. Random variables *X* and *Y* are uniformly distributed on the region bounded by the *x* and *y* axes, and the curve $y = 1 - x^2$.

Calculate E(*XY*).

(A)   0.083
(B)   0.125
(C)   0.150
(D)   0.267
(E)   0.400

225. An insurance company will cover losses incurred from tornadoes in a single calendar year. However, the insurer will only cover losses for a maximum of three separate tornadoes during this timeframe. Let X be the number of tornadoes that result in at least 50 million in losses, and let Y be the total number of tornadoes. The joint probability function for X and Y is

$$p(x, y) = \begin{cases} c(x+2y), & \text{for } x = 0,1,2,3,\ y = 0,1,2,3,\ x \le y \\ 0, & \text{otherwise,} \end{cases}$$

where c is a constant.

Calculate the expected number of tornadoes that result in fewer than 50 million in losses.

(A)   0.19
(B)   0.28
(C)   0.76
(D)   1.00
(E)   1.10

226. At a polling booth, ballots are cast by ten voters, of whom three are Republicans, two are Democrats, and five are Independents. A local journalist interviews two of these voters, chosen randomly.

Calculate the expectation of the absolute value of the difference between the number of Republicans interviewed and the number of Democrats interviewed.

(A)   1/5
(B)   7/15
(C)   3/5
(D)   11/15
(E)   1

227. The random variables X and Y have joint probability function $p(x,y)$ for $x = 0,1$ and $y = 0,1,2$.

Suppose $3p(1,1) = p(1,2)$, and $p(1,1)$ maximizes the variance of XY.

Calculate the probability that X or Y is 0.

(A)   11/25
(B)   23/50
(C)   23/49
(D)   26/49
(E)   14/25

228. As a block of concrete is put under increasing pressure, engineers measure the pressure X at which the first fracture appears and the pressure Y at which the second fracture appears. X and Y are measured in tons per square inch and have joint density function

$$f(x, y) = \begin{cases} 24x(1-y), & 0 < x < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the average pressure (in tons per square inch) at which the second fracture appears, given that the first fracture appears at 1/3 ton per square inch.

(A)   4/9
(B)   5/9
(C)   2/3
(D)   3/4
(E)   80/81

229. The number of severe storms that strike city J in a year follows a binomial distribution with $n = 5$ and $p = 0.6$. Given that m severe storms strike city J in a year, the number of severe storms that strike city K in the same year is m with probability 1/2, $m+1$ with probability 1/3, and $m+2$ with probability 1/6.

Calculate the expected number of severe storms that strike city J in a year during which 5 severe storms strike city K.

(A)   3.5
(B)   3.7
(C)   3.9
(D)   4.0
(E)   5.7

230. Let X denote the proportion of employees at a large firm who will choose to be covered under the firm's medical plan, and let Y denote the proportion who will choose to be covered under both the firm's medical and dental plans.

Suppose that for $0 \le y \le x \le 1$, X and Y have the joint cumulative distribution function

$$F(x, y) = y(x^2 + xy - y^2).$$

Calculate the expected proportion of employees who will choose to be covered under both plans.

(A) 0.06
(B) 0.33
(C) 0.42
(D) 0.50
(E) 0.75

231. Let N denote the number of accidents occurring during one month on the northbound side of a highway and let S denote the number occurring on the southbound side.

Suppose that N and S are jointly distributed as indicated in the table.

$N \setminus S$	0	1	2	3 or more
0	0.04	0.06	0.10	0.04
1	0.10	0.18	0.08	0.03
2	0.12	0.06	0.05	0.02
3 or more	0.05	0.04	0.02	0.01

Calculate Var $(N \mid N + S = 2)$.

(A) 0.48
(B) 0.55
(C) 0.67
(D) 0.91
(E) 1.25

232. An insurance company sells automobile liability and collision insurance. Let X denote the percentage of liability policies that will be renewed at the end of their terms and Y the percentage of collision policies that will be renewed at the end of their terms. X and Y have the joint cumulative distribution function

$$F(x, y) = \frac{xy(x+y)}{2,000,000}, \quad 0 \le x \le 100, 0 \le y \le 100.$$

Calculate Var(X).

(A) 764
(B) 833
(C) 3402
(D) 4108
(E) 4167

233. A hurricane policy covers both water damage, X, and wind damage, Y, where X and Y have joint density function

$$f(x, y) = \begin{cases} 0.13e^{-0.5x-0.2y} - 0.06e^{-x-0.2y} - 0.06e^{-0.5x-0.4y} + 0.12e^{-x-0.4y}, & x > 0, y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the standard deviation of X.

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

234. An insurance company has an equal number of claims in each of three territories. In each territory, only three claim amounts are possible: 100, 500, and 1000. Based on the company's data, the probabilities of each claim amount are:

	Claim Amount		
	100	500	1000
Territory 1	0.90	0.08	0.02
Territory 2	0.80	0.11	0.09
Territory 3	0.70	0.20	0.10

Calculate the standard deviation of a randomly selected claim amount.

(A) 254
(B) 291
(C) 332
(D) 368
(E) 396

235. At the start of a week, a coal mine has a high-capacity storage bin that is half full. During the week, 20 loads of coal are added to the storage bin. Each load of coal has a volume that is normally distributed with mean 1.50 cubic yards and standard deviation 0.25 cubic yards.

During the same week, coal is removed from the storage bin and loaded into 4 railroad cars. The amount of coal loaded into each railroad car is normally distributed with mean 7.25 cubic yards and standard deviation 0.50 cubic yards.

The amounts added to the storage bin or removed from the storage bin are mutually independent.

Calculate the probability that the storage bin contains more coal at the end of the week than it had at the beginning of the week.

(A) 0.56
(B) 0.63
(C) 0.67
(D) 0.75
(E) 0.98

236. An insurance company insures a good driver and a bad driver on the same policy. The table below gives the probability of a given number of claims occurring for each of these drivers in the next ten years.

Number of claims	Probability for the good driver	Probability for the bad driver
0	0.5	0.2
1	0.3	0.3
2	0.2	0.4
3	0.0	0.1

The number of claims occurring for the two drivers are independent.

Calculate the mode of the distribution of the total number of claims occurring on this policy in the next ten years.

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

237. In a group of 15 health insurance policyholders diagnosed with cancer, each policyholder has probability 0.90 of receiving radiation and probability 0.40 of receiving chemotherapy. Radiation and chemotherapy treatments are independent events for each policyholder, and the treatments of different policyholders are mutually independent.

The policyholders in this group all have the same health insurance that pays 2 for radiation treatment and 3 for chemotherapy treatment.

Calculate the variance of the total amount the insurance company pays for the radiation and chemotherapy treatments for these 15 policyholders.

(A) 13.5
(B) 37.8
(C) 108.0
(D) 202.5
(E) 567.0

238. In a large population of patients, 20% have early stage cancer, 10% have advanced stage cancer, and the other 70% do not have cancer. Six patients from this population are randomly selected.

Calculate the expected number of selected patients with advanced stage cancer, given that at least one of the selected patients has early stage cancer.

(A)   0.403
(B)   0.500
(C)   0.547
(D)   0.600
(E)   0.625

239. Four distinct integers are chosen randomly and without replacement from the first twelve positive integers. Let X be the random variable representing the second largest of the four selected integers, and let p be the probability function for X.

Determine $p(x)$, for integer values of x, where $p(x) > 0$.

(A) $\dfrac{(x-1)(x-2)(12-x)}{990}$

(B) $\dfrac{(x-1)(x-2)(12-x)}{495}$

(C) $\dfrac{(x-1)(12-x)(11-x)}{495}$

(D) $\dfrac{(x-1)(12-x)(11-x)}{990}$

(E) $\dfrac{(10-x)(12-x)(11-x)}{990}$

240. An insurance policy covers losses incurred by a policyholder, subject to a deductible of 10,000. Incurred losses follow a normal distribution with mean 12,000 and standard deviation c. The probability that a loss is less than k is 0.9582, where k is a constant. Given that the loss exceeds the deductible, there is a probability of 0.9500 that it is less than k.

Calculate c.

(A)   2045
(B)   2267
(C)   2393
(D)   2505
(E)   2840

241. Losses covered by an insurance policy are modeled by a uniform distribution on the interval [0, 1000]. An insurance company reimburses losses in excess of a deductible of 250.

Calculate the difference between the median and the 20th percentile of the insurance company reimbursement, over all losses.

(A)   225
(B)   250
(C)   300
(D)   375
(E)   500

242. The annual profits that company A and company B earn follow a bivariate normal distribution.

Company A's annual profit has mean 2000 and standard deviation 1000.

Company B's annual profit has mean 3000 and standard deviation 500.

The correlation coefficient between these annual profits is 0.80.

Calculate the probability that company B's annual profit is less than 3900, given that company A's annual profit is 2300.

(A)   0.8531
(B)   0.9192
(C)   0.9641
(D)   0.9744
(E)   0.9953