1st Semester 1436/1437	**Final Exam**	**King Saud University**
(without calculators)	**Time allowed: 3 hours**	**College of Science**
Monday 24-3-1437	**240 Math**	**Math. Department**

Q1: (a) Find the inverse of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$. (4 marks)

(b) Use Cramer's rule to solve the following linear system:

$$\begin{aligned} x_1+2x_3&=6\\ -3x_1+4x_2+6x_3&=30\\ -x_1-2x_2+3x_3&=8 \end{aligned}$$

(4 marks)

Q2: (a) Find a **subset** of the vectors v_1=(1,-2,0,3), v_2=(2,-5,-3,6), v_3=(0,1, 3, 0), v_4=(2,-1,4,-7) and v_5=(5,-8,1,2) that forms a basis for the space **spanned** by these vectors. (4 marks)

(b) Find bases for the eigenspaces of the matrix $A=\begin{bmatrix}3 & 0\\ 8 & -1\end{bmatrix}$ (4 marks)

Q3: (a) Let $\mathbb{R}^4$ have the Euclidean inner product. Find the cosine of the angle θ between the vectors **u**= (4,3,1,-2) and **v**=(-2,1,2,3).

Moreover, if P_2 has the inner product $\langle p,q\rangle=\int_{-1}^{1} p(x)q(x)dx$, then show that the vectors x and x^2 are orthogonal. (4 marks).

(b) Assume that the vector space $\mathbb{R}^3$ has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors (1,1,1), (0,1,1), (0,0,1) into an orthonormal basis. (4 marks)

Q4: (a) Consider the basis S={v_1=(1,1,1),v_2=(1,1,0),v_3=(1,0,0)} for $\mathbb{R}^3$. Let $T:\mathbb{R}^3\to\mathbb{R}^2$ be the linear transformation for which T(v_1)=(1,0), T(v_2)=(2,-1) and T(v_3)=(4,3). Find a formula for T(x_1,x_2,x_3), and then use that formula to compute T(2,-3,5). (4 marks)

(b) Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be the linear operator defined by the formula:

$$T(x_1,x_2,x_3)=(3x_1+x_2,-2x_1-4x_2+3x_3,5x_1+4x_2-2x_3)$$

Determine whether T is one-to-one; if so, find $T^{-1}(x_1, x_2, x_3)$. (4 marks)

Q5: Prove the following statements:
(a) If **u** and **v** are orthogonal vectors in an inner product space, then
$\|u+v\|^2 = \|u\|^2 + \|v\|^2$. (2 marks)
(b) If $T : V \to W$ is a linear transformation, then the range of T is a subspace of W. (2 marks)
(c) If $T : V \to W$ is a linear transformation, then T is one-to-one if and only if ker(T)={0}. (2 marks)
(d) If $T_1 : U \to V$ and $T_2 : V \to W$ are two linear transformations, then
$(T_2 \circ T_1) : U \to W$ is also a linear transformation. (2 marks)