First Semester	**Final Exam**	**King Saud University**
(without calculators)	**Time allowed: 3 hours**	**College of Science**
Monday 2-7-1447	**240 Math**	**Math. Department**

Q1: Solve the following system:

$$x_1 + x_2 - x_3 = 1$$
$$x_2 - 3x_3 = 1$$
$$x_3 = 1$$

(i) by Gauss-Jordan elimination. (3 marks)

(ii) Let A be the coefficient matrix of the system. Find det(A) and A^{-1}. (5 marks)

Q2: Using the following matrix, find (6 marks)

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 5 & 6 & 1 \\ 1 & 3 & 4 & 2 \end{bmatrix}$$

(i) the basis of the column space of A.

(ii) the basis of the null space of A.

(iii) nullity(A^T).

Q3: Let W={(2x,0,3y,0)|x,y∈ℝ}.

(i) Show that W is a **subspace** of $\mathbb{R}^4$. (3 marks)

(ii) Find a basis of W. (2 marks)

(iii) **Find** dim(W). (1 mark)

Q4: If $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix}$, then

(i) find its eigenvalues. (1 mark)

(ii) Find the matrix P that diagonalizes A. (4 marks)

Q5: Let P_2 be the vector space of all polynomials of degree less than or equal to 2 with the standard inner product.

(i) Compute $\langle 1, x^2 \rangle$. (1 mark)

(ii) Let $\{p_1=1+x+x^2, p_2=2-x+2x^2\}$ be a basis of a subspace of P_2. Apply the Gram-Schmidt process to transform that basis into an **orthonormal basis**. (4 marks)

Q6: Take V={$A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$ |$a \in \mathbb{R}$} which is a subspace of M_{22} and let T:V→ℝ be the map defined by T(A)=a for all matrices A in V. Show that:

(i) T is a linear transformation. (2 marks)

(ii) Find a basis of ker(T). (1 mark)

(iii) Find $[T]_{S,B}$ where S={1} and B={$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$}. (2 marks)

(iv) Find rank(T). (1 mark)

Q7: (i) If B={(1,2),(2,3)} is a basis of $\mathbb{R}^2$ and $(u)_B=(1,2)$, then find u. (1 mark)

(ii) If A=$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, then why A couldn't be a transition matrix. (1 mark)

(iii) If u and v are orthogonal non-zero vectors in an inner product space V, then show that 2u and 3v are linearly independent. (2 marks)