

Question (1): An insurance sales representative sells policies to 5 men, all of identical age and in good health. According to the actuarial tables, the probability that a man of this particular age will be alive 30 years hence is 0.6. $p = 0.6$ $q = 0.4$ $n = 5$

- a) The probability that in 30 years at least 1 man will be alive is

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) \\ = 1 - \left[\binom{5}{0} (0.6)^0 (0.4)^5 \right] \\ = 1 - 0.01024 = 0.9898$$

- b) The expected number of men who will be alive 30 years is

$$E(X) = np = 5 \times 0.6 = 3$$

- c) The standard deviation of the number of men who will be alive 30 years is

$$\text{Var}(X) = npq = 5 \times 0.6 \times 0.4 = 1.2$$

$$\text{S.D.} = \sqrt{1.2} = \frac{\sqrt{30}}{5}$$

Question (2): If the heights (X) of students are normally distributed with mean 68.0 inches and standard deviation 3.0 inches, $\mu = 68$ $\sigma = 3$

- a) The probability of students height who have between 65 and 71 inches inclusive is

$$P(65 \leq X \leq 71) = P\left(\frac{65 - 68}{3} \leq Z \leq \frac{71 - 68}{3}\right) \\ = P(-1 \leq Z \leq 1) = P(Z \leq 1) - P(Z \leq -1) \\ = 0.8413 - 0.15866 = 0.68264$$

- b) If $P(X < k) = 0.975$, $k =$?

$$\boxed{-\frac{1}{2}}$$

$$\frac{k - 68}{3} = 1.96$$

- c) The probability of student height who have exactly the median is

$$P(68 \leq X) = 0.5$$

$$P\left(\frac{X - 68}{3} \leq x\right) = 0.5$$

$$P(-)$$

$$\frac{X - 68}{3} = 0$$

$$\boxed{-\frac{1}{2}}$$

$$\boxed{X = 68}$$

$$N = 15 \begin{matrix} \rightarrow 9 \text{ H} \\ \rightarrow 6 \text{ D} \end{matrix}$$

$$n = 4 \quad K = 9$$

Question (3): In a list of 15 households, 9 own homes and 6 do not own homes. Four households are randomly selected from these 15 households. Find

- a) the probability that the number of households in these 4 who own homes is exactly 3.

$$N = 15, n = 4, m = 9, X = 3$$

$$P(X=3) = \frac{\binom{9}{3} \binom{15-9}{4-3}}{\binom{15}{4}} = \frac{\binom{9}{3} \binom{6}{1}}{\binom{15}{4}} = \frac{24}{65} = 0.3692$$

- b) the probability that the number of households in these 4 who own homes is at most 1.

$$P(X \leq 1) = P(X=0) + P(X=1)$$

$$= \frac{\binom{9}{0} \binom{6}{4}}{\binom{15}{4}} + \frac{\binom{9}{1} \binom{6}{3}}{\binom{15}{4}} = \frac{1}{91} + \frac{12}{91} = \frac{1}{7}$$

- c) The standard deviation of the number of households in these 4 who own homes.

$$\begin{aligned} \text{Var}(X) &= n \frac{M}{N} \left(1 - \frac{M}{N}\right) \frac{N-n}{N-1} \\ &= 4 \left(\frac{9}{15}\right) \left(1 - \frac{9}{15}\right) \frac{15-4}{15-1} = \frac{36}{15} \left(\frac{2}{5}\right) \frac{11}{14} = \frac{132}{175} \\ S.D. &= \sqrt{\frac{132}{175}} = 0.8685 \end{aligned}$$

Question (4): Suppose X is a continuous random variable with uniform distribution having mean 1 and variance $\frac{3}{4}$. $\mu = 1 \quad \sigma^2 = \frac{3}{4}$

- a) The pdf of X is

$$E(X) = \frac{b+a}{2} \Rightarrow 1 = \frac{b+a}{2} \Rightarrow b+a=2$$

$$\text{Var}(X) = \frac{(b-a)^2}{12} \Rightarrow \frac{3}{4} \Rightarrow (b-a)^2 = 9 \Rightarrow (b-a) = \pm 3$$

$$f(x) = \frac{1}{5/2 - (-1/2)} = \frac{1}{3} \quad -0.5 < X < 2.5$$

- b) $P(X < 0) =$

$$F(0) = \frac{x-a}{b-a} = \frac{0 - (-0.5)}{2.5 - (-0.5)}$$

$$= \frac{0.5}{3} = \frac{1}{6}$$

$$b = 2 - a$$

$$a = -0.5$$

$$b = 2.5$$

$$b - a = 3$$

$$4.2 = 2a + 3$$

$$1.2 = 2a$$

$$a = 0.6$$

$$b = 1.4$$

$$b = \frac{5}{2} = 2.5$$

Question (5): Let X be the number of fatal car accidents in a certain city per a day. The average of fatal car accidents per a day is one. $\lambda = 1 \rightarrow$ per day

- a) The probability of more than 2 accidents in a week.

$$P(2 < X) = 1 - P(X \leq 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\sum_{x=0}^2 \frac{e^{-\lambda} \lambda^x}{x!} \right] = 1 - 0.0296$$

$$= 0.9704$$

$\lambda \rightarrow$ day
1 $\rightarrow$ 1
? $\rightarrow$ 7
 $\lambda = 7$

$$\frac{e^{-\lambda} \lambda^x}{x!}$$

- b) The mean of number of accidents in 3 days.

$\lambda \rightarrow$ day
1 $\rightarrow$ 2
? $\rightarrow$ 3
 $\lambda = 3 = E(X)$

5W
2R 4B

$$N = 10$$

$$P(R) = \frac{1}{10}$$

Question (6): An urn contains 5 white, 4 black, and 1 red marble. Marbles are drawn, with replacement, until a red one is found. If X is the random variable counting the number of trials until the first red marble appears,

- a) The probability that the first red marble appears on the second trial is $r = \frac{1}{10} \rightarrow$ Geometric
 $X = 2$

$$P(X=2) = P(1-P)^{X-1}$$

$$= \frac{1}{10} \left(\frac{9}{10} \right)^{2-1} = \frac{9}{100} = 0.09$$

b) $P(X=4|X \geq 1) = P(X \leq 3) = 1 - P(X \leq 3) = 1 - [P(X=1) + P(X=2) + P(X=3)]$

$$= 1 - \left[\sum_{x=1}^3 \left(\frac{1}{10} \right) \left(\frac{9}{10} \right)^{x-1} \right]$$

$$= 1 - 0.271 = 0.729$$

$$\frac{3}{4}$$

c) $E(2X^2 + 1) =$

$$2E(X^2) + 1$$

$$= 2(190) + 1$$

$$= 381$$

$$Var(X) = E(X^2) - E(X)^2$$

$$\frac{0.9}{(0.1)^2} = E(X^2) - \left(\frac{1}{0.1} \right)^2$$

$$90 = E(X^2) - 100$$

$$E(X^2) = 190$$

$$E(X) = \frac{1}{p}$$

$$Var = \frac{q}{p^2}$$

Question (7): A comedian tells jokes that are only funny 25% of the time.

$$p = 0.25$$

$$q = 0.75$$

- a) The probability that he tells his 10-th funny joke in his 40-th attempt is

$$p(x=40) = \binom{40-1}{10-1} (0.25)^{10} (0.75)^{30}$$

$$= 0.03609$$

- b) The expected value of the attempts of the comedian to tell his 6-th funny joke is

$$E(x) = \frac{r}{p} = \frac{6}{0.25} = 24$$

Question (8): The distance between major cracks in a highway follows an exponential distribution with a mean of 5 miles.

$$\lambda = \frac{1}{5}$$

- a) The probability that the first major crack occurs between 15 and 20 miles of the start of inspection is

$$P(15 \leq x \leq 20)$$

$$= P(x \leq 20) - P(x \leq 15)$$

$$= (1 - e^{-\frac{20}{5}}) - (1 - e^{-\frac{15}{5}})$$

$$= e^{-3} - e^{-4} = 0.0315$$

$$F(x) = 1 - e^{-\lambda x}$$

- b) Given that there are cracks after 5 miles inspected, the probability that there are major cracks after 20 miles is

$$P(x \geq 20 | x \geq 5) = \frac{P(x \geq 20 \cap x \geq 5)}{P(x \geq 5)} = \frac{P(x \geq 20)}{P(x \geq 5)} = \frac{e^{-\frac{20}{5}}}{e^{-\frac{5}{5}}} = \frac{e^{-4}}{e^{-1}} = e^{-3} = 0.04978$$

$$\downarrow$$

$$\frac{1 - P(x < 20)}{1 - P(x < 5)} = \frac{1 - (1 - e^{-\frac{20}{5}})}{1 - (1 - e^{-\frac{5}{5}})}$$

- c) Let $Y = \min(Y_1, \dots, Y_5)$, where $Y_1, \dots, Y_5$ are independent exponential distributed with mean $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{3}$ and 1, respectively. The distribution of Y is ~~exponential~~

$$\mu = \frac{1}{\lambda}$$

$$\frac{1}{2} = \frac{1}{\lambda}$$

$$\lambda = 2$$

$$Y = (\lambda_1, \lambda_2, \lambda_3, \dots)$$

$$Y = (2, 3, 5, 3, 1)$$

Parameter λ

$$\lambda = (2, 3, 5, 3, 1) = 14$$

$$\mu = \frac{1}{14}$$

Question (9): let X be a random variable with density function as follows

$$f(x) = \frac{1}{4}e^{-x} + \frac{1}{2}e^{-2x} + 2e^{-4x}, x > 0.$$

a) The distribution of X is ~~mixed dist~~ $\rightarrow$ $\frac{1}{4} \cdot 8$
 $\rightarrow$ expo

b) $\text{Var}(X - 1) =$

c) $M_X(t) =$

Question (10): Suppose that X has a discrete uniform distribution on 50 points. $N=50 > 30$

a) The approximate probability $P(10 \leq X \leq 30) =$

$$E(x) = \frac{50+1}{2} = 25.5$$

$$\text{Var} = \frac{(50)^2 - 1}{12} = 208.25$$

$$\sigma = 14.431$$

$$\boxed{-\frac{1}{4}}$$

$$P(10 - \frac{1}{2} \leq x \leq 30 + \frac{1}{2})$$

$$= P(9.5 \leq x \leq 30.5)$$

$$= P\left(Z \leq \frac{30.5 - 25.5}{14.431}\right) - P\left(Z \leq \frac{9.5 - 25.5}{14.431}\right)$$

$$= P(Z \leq 0.6929) - P(Z \leq -1.1087)$$

$$= 0.7558 - 0.1338 = 0.622$$

End of Questions