Question (1): The probability that a man will hit a target is 3/4. If he shoots at the target until he hits it for the first time,

$\frac{3}{4}$

$p \cdot 25$

a) The probability that it will take him **5 shots** to hit the target is.

$1 \rightarrow 4$

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$$p \quad \left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) \times \left(\frac{3}{4}\right) = \frac{3}{1024} = 2.92 \times 10^{-3}$$

$$* p(\text{hit}T) = 0.75 \qquad p(\text{hit}T)^c = 0.25$$

Question (2): **Three balls** are drawn successively from a box containing **6 red** balls, **4 white** balls, and **5 blue balls**. $T = 15$

$R = 6, \quad W = 4, \quad B = 5$

a) The probability that they are drawn in the order **red, white,** and **blue** if each ball is not replaced is.

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$$\frac{n!}{(n-k)!}$$

$$\frac{6}{15} \times \frac{4}{14} \times \frac{5}{13} = \frac{4}{91}$$

Question (3): **Box I** contains **3 red** and **2 blue** marbles while **Box II** contains **2 red** and **8 blue** marbles. A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box I; if it turns up tails, a marble is chosen from Box II.

$\boxed{3R \; 2B}\ 5$ I B_1 $\qquad \boxed{2R \; 8B}\ 10$ II B_2

$B_1 = P$ for first Box

$B_2 = P$ for Second Box

1.5

a) The probability that a **red** marble is chosen is.

$$P(R) = P(R|B_1)P(B_1) + P(R|B_2)P(B_2)$$

$$= \left(\frac{3C_1}{5C_1}\right)\left(\frac{1}{2}\right) + \left(\frac{2C_1}{10C_1}\right)\left(\frac{1}{2}\right) = \frac{2}{5}$$

Coin

tail $\frac{1}{2}$ $\qquad$ Head $\frac{1}{2}$

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b) **If a red** marble is chosen, the probability that **Box I** was chosen is.

$$P(B_1|R) = \frac{P(R|B_1)P(B_1)}{P(R) \rightarrow P(R|B_1)P(B_1) + P(R|B_2)P(B_2)}$$

$$= \frac{3/10}{2/5} = \frac{3}{4}$$

Question (4): Five **red** marbles, **two white** marbles, and **three blue** marbles are arranged in a row. If all the marbles of the same color are not distinguishable from each other, how many different arrangements are possible?

1.25

$$\text{ToTal} \leftarrow \frac{n!}{n_1!\, n_2!\, n_3!} = \frac{10!}{5!\,2!\,3!} = 2520$$
$$\text{Types} \leftarrow$$

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Question 5: Out of **5 mathematicians** and **7 physicists**, a committee consisting of **2 mathematicians** and **3 physicists** is to be formed. In how many ways can this be done if one particular physicist must be on the committee.

1.25

$$nC_k = \frac{n!}{k!(n-k)!}$$

$$5C_2 \cdot 6C_2 = 150$$

Question (6): If A and B are independent and $P(A) = P(B|A) = 0.3$,

$P(B) = 0,3$
↳ Since It's Indepen

$P(B \cap A) = 0,3 \times 0,3$
$= 0,09$

1.5 a) $P(A \cup B) = P(B) + P(A) - P(B \cap A)$

$= 0,3 + 0,3 - 0,09 = 0,51$

1.5 b) $P(B^c|A^c) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = \frac{P(\overline{B \cup A})}{1 - P(A)} = \frac{1 - P(B \cup A)}{1 - P(A)}$

$= \frac{1 - 0,51}{1 - 0,3} = \frac{7}{10}$

Question (7): The distribution function for a random variable X is

$$F(x) = \begin{cases} 1 - e^{-2x}, & x \geq 0 \\ 0, & x < 0. \end{cases}$$

1 a) The density function $f(x)$ is

$F'(x) = f(x)$

$= -(-2)\, e^{-2x} = 2e^{-2x}, \quad x > 0$

b) $P(-3 < X < 4) =$ $P(0<x<4)$

$\int_0^4 2e^{-2x}\,dx = [-e^{-2x}]_0^4 = -e^{-2(4)} - (-e^{-2(0)})$ or $F(4) - F(0)$

$= -e^{-8} + e^{0} = 1 - e^{-8}$

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Question (8): The probability function of a random variable X is given by

$$f(x) = \begin{cases} p & x = 1 \\ 3p & x = 2 \\ 2p & x = 3 \\ 0 & o.w, \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{6} & x = 1 \\ \frac{1}{2} & x = 2 \\ \frac{1}{3} & x = 3 \\ 0 & o.w \end{cases}$$

1 **a)** $p =$ $P + 3P + 2P = 1 \implies 6P = 1 = P = \frac{1}{6}$

1 **b)** $M(t) =$ $E(e^{tx}) = \sum_{x=1}^{3} e^{tx} f(x) = e^{t}\frac{1}{6} + e^{2t}\frac{1}{2} + e^{3t}\frac{1}{3}$

$= \frac{e^t}{6} + \frac{3e^{2t}}{6} + \frac{2e^{3t}}{6} = \frac{e^t + 3e^{2t} + 2e^{3t}}{6}$

1 **c)** $P(X \geq 2.5) =$ $P(x=3) = \frac{1}{3}$

$P(x = 2,5) = 0$

1 **d)** $Var(3X - 2) =$ $9\,Var(x) = E(x^2) - (E(x))^2$

$E(x) = \sum x f(x) = 1 \times \frac{1}{6} + 2 \times \frac{1}{2} + 3 \times \frac{1}{3} = \frac{1}{6} + 1 + 1 = \frac{13}{6}$

$E(x^2) = \sum x^2 f(x) = 1 \times \frac{1}{6} + 4 \times \frac{1}{2} + 9 \times \frac{1}{3} = \frac{1}{6} + 2 + 3 = \frac{31}{6}$

$9\,var(x) = 9\left(\frac{31}{6} - \frac{169}{36}\right) = 9 \times \frac{17}{36} = \frac{17}{4}$

Question (9): Let X be a random variable having density function

$$f(x) = \begin{cases} 1/4, & -2 \le x \le 2 \\ 0, & o.w. \end{cases}$$

a) The median is $P(X \le M) = 0,5$

$\int_{-2}^{M} \frac{1}{4}\,dx = \frac{1}{4}x\Big]_{-2}^{M} = \frac{M}{4} + \frac{2}{4} = \frac{M}{4} + \frac{1}{2} = 0,5$

$\frac{M}{4} = 0 \Rightarrow \boxed{M = 0}$

b) The Expected value of $(X^3 - 1)$ is

$E(x^3-1) = \int_{-\infty}^{\infty} (x^3-1)\, f(x)\, dx$

$= \int_{-2}^{2} (x^3-1)\frac{1}{4}\,dx = \frac{1}{4}\int_{-2}^{2} (x^3-1)\,dx = \frac{1}{4}\left[\frac{x^4}{4} - x\right]_{-2}^{2} = \frac{1}{4}\left[\left(\frac{16}{4}-2\right)-\left(\frac{16}{4}+2\right)\right]$

$= \frac{1}{4}(2-6) = \frac{-4}{4} = -1$

c) The 90-th percentile of the distribution of X is

$P(X \le C) = 0,9$

$\int_{-2}^{C} \frac{1}{4}\,dx = 0,9 \Rightarrow \frac{x}{4}\Big]_{-2}^{C} = 0,9 \Rightarrow \frac{C}{4} + \frac{1}{2} = 0,9$

$\frac{C}{4} = 0,4$

$C = \frac{8}{5} = 1,6$

Question (10): A random variable X has mean 3 and variance 2, what is $P(|X - 3| \ge 2)$?

$E(x) = 3 \quad Var(x) = 2$

$P(|X-\mu| > r) \geq \frac{6^2}{r^2}$

$P(|X-3| \geq 2) \geq \frac{2}{4}$

$P(|x-3| \geq 2) \geq \frac{1}{2}$

Question (11): Give an example of a random variable with mean 0 and variance 1.

X	f(x)
1	0,5
-1	0,5

$f(x) = \begin{cases} 0,5, & x = 1 \\ 0,5, & x = -1 \\ 0, & o.w. \end{cases}$

$E(x) = 0,5 \times 1 + 0,5(-1) = 0$

$Var(x) = (1)^2 \times 0,5 + (-1)^2 \times 0,5 = 1$

Question (12): Let X is a random variable with moment generated function $M(t) = q + pe^t$ for $-\infty < t < \infty$, $p + q = 1$. The standard deviation is

$E(x) = M'_x(t) = e^t p \big|_{t=0} = e^0 p = p$

$E(x^2) = M''_x(t) = e^t p \big|_{t=0} = e^0 p = p$

$Var = E(x^2) - (E(x))^2 = p - p^2$

$= p(1-p) \Rightarrow pq$

$\sigma = \sqrt{\sigma^2} = \sqrt{pq}$

1.5

End of Questions