1

اختبارات نهائية
Page (236 - 234)

اختبار رقم 1

$f(x) = \frac{x}{c}$ $\quad x = 2, 3, 5$

1. find value of c :-

x	2	3	5
$f(x)$	$\frac{2}{c}$	$\frac{3}{c}$	$\frac{5}{c}$

$$\sum f(x) = 1$$

$$\frac{2}{c} + \frac{3}{c} + \frac{5}{c} = 1.$$

$$\frac{10}{c} = 1 \rightarrow \boxed{c = 10}$$

2. Find $F(x)$ = Cumulative prob.

$$F(x) = p(X \le x) = \sum_{x=2}^{y} f(x) = \cancel{Z}$$

$$\cancel{\frac{2}{10} + \frac{3}{10} + \frac{5}{10} + \frac{y}{10}}$$

x	2	3	5
$f(x)$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{5}{10}$
$F(x)$	$\frac{2}{10}$	$\frac{5}{10}$	1

$$f(x) = \begin{cases} 0 & x < 2 \\ \frac{2}{10} & 2 \le x < 3 \\ \frac{5}{10} & 3 \le x < 5 \\ 1 & 5 \le x \end{cases}$$

3. $M_x(t) = E(e^{tx}) = \sum_{x=2}^{5} e^{tx} f(x)$

$$= \frac{2}{10} e^{2t} + \frac{3}{10} e^{3t} + \frac{5}{10} e^{5t}$$

$$= 0.2 e^{2t} + 0.3 e^{3t} + 0.5 e^{5t}$$

2

اختبار زماني ①

④ $M_Y(t) = \cancel{E} M_{2X-1}(t)$

$= e^{-t} M_X(2t)$

$= e^{-t}\left[0.2e^{4t} + 0.3e^{6t} + 0.5e^{10t}\right]$

$= 0.2e^{3t} + 0.3e^{5t} + 0.5e^{9t}$

5 $f(y) = ??$

$x = 2, 3, 5$

$y = 3, 5, 9$ ← تعويض عن X $y = 2x-1$

$y = 2X - 1$

$f^{-1}(y) = x = \frac{y+1}{2}$ one-to-one

$f(x) = \frac{x}{10}$

$f_X(y) = f_X(f^{-1}(y)) = f_X\left(\frac{y+1}{2}\right) = \frac{1}{10}\left(\frac{y+1}{2}\right)$

$\boxed{f(y) = \frac{y+1}{20} \qquad y = 3, 5, 9}$

(ب) $\mu'_r = E(X^r) = \frac{r!}{2!}$

توقع $= E(X) = \mu'_1 = E(X^1) = \frac{1!}{2!} = \frac{1}{2}$

$var(X) = E(X^2) - [E(X)]^2$

نجد $\boxed{E(X^2) \rightarrow r = 2}$

$E(X^2) = \frac{r!}{2!} = \frac{2!}{2!} = 1$

$\therefore var(X) = E(X^2) - [E(X)]^2$

$= 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} = 0.75$

السؤال الثاني

③ اختبار نهائي

Find

① من Moments $M_x(t)$ نتعرف على اسم التوزيع ثم نكتب $E(x)$ $V(x)$

(5) $M_x(t) = \left(\frac{1}{4}\right)^{10}\left(3e^t+1\right)^{10} = \left(\frac{3}{4}e^t+\frac{1}{4}\right)^{10}$

Binomail $\Rightarrow (pe^t+q)^n$

$n=10,\ p=\frac{3}{4},\ q=\frac{1}{4}$

$E(x) = np = 10 \times \frac{3}{4} = 10 \times 0.75 = 7.5$

$V(x) = npq = 10 \times \frac{3}{4} \times \frac{1}{4} = 1.875$

---

(1) $M_x(t) = \frac{e^{3t}-e^{2t}}{t} \Longleftrightarrow \frac{e^{bt}-e^{at}}{(b-a)t}$ (continuous Uniform $(2,3)$ $a=2,\ b=3$

$E(x) = \frac{a+b}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$

$V(x) = \frac{(b-a)^2}{12} = \frac{(3-2)^2}{12} = \frac{1}{12}$

---

(3) $M_x(t) = \left(\frac{4}{4-t}\right)^2 \Longleftrightarrow \left(\frac{\beta}{\beta-t}\right)^{\alpha}$ [gamma distribution $\alpha=2,\ \beta=4$]

$E(x) = \frac{\alpha}{\beta} = \frac{2}{4} = \frac{1}{2}$, $V(x) = \frac{\alpha}{\beta^2} = \frac{2}{16} = \frac{1}{8}$

---

(2) $M_x(t) = (1-2t)^{-8} \Longleftrightarrow (1-2t)^{-v/2}$ [chi-square $\frac{v}{2}=8 \Rightarrow v=16$

$E(x) = v = 16$, $V(x) = 2v = 2*16 = 32$

---

④ $M_x(t) = e^{\frac{1}{2}t^2}$ [Standard normal $Z \sim N(0,1)$]

$E(x) = \mu = 0$, $V(x) = \sigma^2 = 1$

السؤال الثاني (ب)

4

اختبار نهائي

1

$X \sim Poisson(\lambda)$

$$f(1) = f(2)$$

$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

λ مجهول لايجادها نفك المعادلة

$$\frac{\cancel{e^{-\lambda}}\lambda^1}{1!} = \frac{\cancel{e^{-\lambda}}\lambda^2}{2!}$$

$$\lambda = \frac{\lambda^2}{2!}$$

$$2 = \frac{\lambda^{\cancel{2}}}{\cancel{\lambda}} = \lambda \Rightarrow \boxed{\lambda = 2}$$

$$\boxed{f(3)} = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-2}\,2^3}{3!} = \boxed{0.18044}$$

$$M_x(t) = e^{\lambda(e^t-1)}$$

$$= e^{2(e^t-1)}$$

ع.

السؤال الثالث

5

اختبار ثماني

1

$$f(x,y) = 4xy \quad 0 < x, y < 1$$

$$f_x(x) = \int_0^1 f(x,y)\,dy = \int 4xy\,dy = 4x\frac{y^2}{2}\Big|_0^1$$

$$= \cancel{2}\,x(1^2-0) = 2x$$

$$f_y(y) = \int_0^1 f(x,y)\,dx = \int_0^1 4xy\,dx = 4\frac{x^2}{2}y\Big|_0^1$$

$$= 2y(1^2-0) = \boxed{2y}$$

$$E(x) = \int_0^1 x f(x)\,dx = \int_0^1 x(2x)\,dx = \int 2x^2\,dx$$

$$= 2\frac{x^3}{3}\Big|_0^1 = \frac{2}{3}$$

$$E(y) = \cancel{2}\int_0^1 y f(y)\,dy = \boxed{\frac{2}{3}}$$

$$E(xy) = \int_0^1\int_0^1 xy f(x,y)\,dx\,dy = \int_0^1\int_0^1 xy(4xy)\,dx\,dy$$

$$= \int_0^1\int_0^1 4x^2y^2\,dx\,dy = \int_0^1 4\frac{x^3}{3}y^2\Big]_0^1\,dy$$

$$= \int_0^1 \frac{4}{3}y^2\,dy = \frac{4}{3}\frac{y^3}{3}\Big|_0^1 = \boxed{\frac{4}{9}}$$

الاختبار الثاني : السؤال الرابع

		-2	0	5
x	1	0.15	0.25	0.2
	3	c	0.05	0.15

1) Find C

$$\sum\sum f(x,y) = 1$$

$$0.15 + 0.25 + 0.2 + C + 0.05 + 0.15 = 1$$

$$0.8 + C = 1$$

$$C = 1 - 0.8$$

$$\boxed{C = 0.2}$$

2] $$f_{x/y}(1/0) = \frac{f(x,y)}{f_y(y)}$$

$$= \frac{f(1,0)}{f_y(0)} = \frac{0.25}{0.25+0.05} = \frac{0.25}{0.3} = 0.833$$

3) $$F_{x/y}(1,5) = p(X \leq 1 / y=5) = \frac{f(X \leq 1, Y=5)}{f_y(5)}$$

$$= \frac{f(1,5) \cancel{+ f(3,5)}}{0.2 + 0.15} = \frac{0.15}{0.35} = 0.4285$$

4) $$f_Z(z) = f_{x+y}(x,y) =$$

Z	-1	1	3	6	8
	$f(1,-2)$ 0.15	$f(1,0) + f(3,-2)$ 0.25 + 0.2 = 0.45	$f(3,0)$ 0.05	$f(1,5)$ 0.2	$f(3,5)$ 0.15

$Z = X + Y$

$1 - 2 = -1$

$1 + 0 = 1$

$1 + 5 = 6$

$Z = X + Y$

$3 - 2 = 1$

$3 + 0 = 3$

$3 + 5 = 8$

Z	-1	1	3	6	8	Σ
$f(Z)$	0.15	0.45	0.05	0.2	0.15	1

7

اهيب زهران

1

السؤال الرابع (ب)

$$f_{X_1,X_2}(x_1,x_2) = e^{-(x_1+x_2)} \qquad 0 \le x_1, x_2 \le \infty$$

$$Y_1 = X_1 + X_2 \qquad Y_2 = \frac{X_1}{X_1+X_2}$$

$$Y_1 = Y_1 Y_2 + X_2 \qquad Y_2 = \frac{X_1}{Y_1}$$

$$X_2 = Y_1 - Y_1 Y_2$$

$$X_2 = Y_2(1 - Y_1) \qquad \boxed{X_1 = Y_1 Y_2}$$

$$\boxed{0 < y_1 < 1} \qquad \boxed{0 < y_2 < \infty}$$

$$g_1^{-1}(y_1,y_2) = y_1 y_2 \qquad g_2^{-1}(y_1,y_2) = y_2(1-y_1)$$

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix} = \begin{vmatrix} y_2 & y_1 \\ 1-y_2 & -y_1 \end{vmatrix} = -y_1 y_2 - y_1(1-y_2)$$

$$= -y_1$$

$$f_{Y_1,Y_2}(y_1,y_2) = |J|\, f_{X_1,X_2}\left(g_1^{-1}(y_1,y_2),\, g_2^{-1}(y_1,y_2)\right)$$

$$= |-y_1|\, e^{-(y_2(1-y_1) + y_1 y_2)}$$

$$= y_1\, e^{-(y_2 - \cancel{y_1 y_2} + \cancel{y_1 y_2})} = y_1 e^{-y_2}$$

⑧

اقتباس دراسات

①

$$f_{Y_2}(y_2) = \int f(y_1, y_2)\, dy_1$$

$$= \int y_1 e^{-y_2}\, dy_1 = \int_0^1 y_1 e^{-y_2}\, dy_1$$

$$= e^{-y_2} \frac{y_1^2}{2}\Big|_0^1 = \frac{e^{-y_2}}{2}$$

$$f_{y_1}(y_1) = \int f(y_1, y_2)\, dy_2 = \int_0^\infty y_1 e^{-y_2}\, dy_2$$

$$= -y_1 e^{-y_2}\Big|_0^\infty = y_1(1 - \cancel{e^{-\infty}}) = y_1$$