King Saud University — Department of Mathematics
Final Exam — MATH 205: Questions and Solutions

Semester II, 2025/26 Marks: 40 Time: 3 Hours

Q1. [3+3+5+3+3 Marks]

1. Discuss the convergence of the sequence
$$\left\{\frac{\sin n}{n^2}\right\}_{n=1}^{\infty}.$$
2. Determine whether the series
$$\sum_{n=1}^{\infty}\frac{(-1)^n}{e^n}$$
is absolutely convergent, conditionally convergent, or divergent.
3. Find the radius and interval of convergence of
$$\sum_{n=0}^{\infty}\frac{(-1)^n(x+1)^n}{n+1}.$$
4. Use the first two nonzero Maclaurin terms to approximate
$$\int_0^{0.5} x\cos(x^2)\,dx.$$
5. Evaluate
$$\int_0^1\int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\,dy\,dx.$$

Q2. [3+3 Marks]

a) Given
$$P(1,-1,-1),\quad Q(1,1,-1),\quad R(2,-1,1),$$
find:
  i) A unit vector perpendicular to plane PQR.
  ii) Area of triangle PQR.
  iii) Distance from R to the line through P, Q.

b) Find the equation of the intersection of the planes
$$x+y-z=2,\qquad 2x-y+3z=1.$$

Q3. [3+3+3 Marks]

a) Use the chain rule to find
$$\frac{\partial z}{\partial s},\quad \frac{\partial z}{\partial t},$$
where
$$z=e^x\cos y,\quad x=st^2,\quad y=s^2t.$$

b) Evaluate
$$\int_0^1\int_x^1 2\cos(y^2)\,dy\,dx.$$

c) Determine whether the limit exists:
$$\lim_{(x,y)\to(0,0)}\frac{x^2y}{x^4+y^2}.$$

Q4. [4+4 Marks]

a) Find local extrema and saddle points of
$$f(x,y)=x^3+y^3-3x-12y+20.$$

b) Use Lagrange multipliers to find extrema of
$$f(x,y)=x^2-y^2,$$
subject to
$$x^2+y^2=1.$$