King Saud University, Department of Mathematics
Final Exam, Second Term, 2024/25, Course: M-205

Marks: 40, Time: 3 hours

Question 1 [3+3+5+3+3 Marks]

1. Discuss the convergence of the sequence $\sum_{n=1}^{\infty}\frac{3^n}{10^2+2^n}$.
2. Determine whether the series $\sum_{n=1}^{\infty} ne^{-n^2}$ converges or diverges.
3. Find the radius and the interval of convergence of the power series $\sum_{n=0}^{\infty}\frac{(x-1)^n e^n}{n+1}$.
4. Use the first three non-zero terms of ta Maclaurin series to approximate the integral $\int_0^{0.1}\frac{\sin(x^2)}{x^2}\,dx$.
5. Evaluate the double integral $\int_0^{\frac{\pi}{4}}\int_y^{\frac{\pi}{4}}\frac{x\sin x}{x^2+y^2}\,dxdy$.

Question 2 [3+3 Marks]

a) Find the domain of the function $f(x,y)=\sqrt{\frac{x^2+y^2}{x^2-y^2}}$ and find $\lim_{(x,y)\to(0,0)} f(x,y)$.

b) Find the area of the surface $x^2+y^2+z^2=25$ that lies above the plane $z=4$.

Question 3 [3+3+3 Marks]

a) Use Chain rule to find $\frac{\partial w}{\partial x},\frac{\partial w}{\partial y}$ where $w=r^2+s^2, r=x-y$ and $s=x+y$.

b) Find the points on the paraboloid $z=x^2+y^2$ at which the normal line is parallel to the line through the points $A(1,-1,0)$ and $B(0,1,1)$.

c) Find the directional derivative of $f(x,y,z)=xze^y+\cos(xy)$ at the point $P(2,0,1)$ in the direction of the line $x=-1+3t, y=2-4t, z=1+5t$. In which direction it increases most rapidly? What is the maximum rate of increase of f at P?

Question 4 [4+4 Marks]

(a) Find local extrema and saddle points, if any, on the surface

$$f(x,y)=x^3-y^2-3x+2y.$$

(b) Use method of Lagrange multiplier to find extrema of the function $f(x,y)=x+2y$, subject to constraint $x^2+y^2=1$.