**King Saud University, Department of Mathematics**
**Final Exam, First Term, 2024/25, Course: M-205**

Marks: 40, Time: 3 hours

**Question 1 [3+3+5+3+3 Marks]**

1. Discuss the convergence of the sequence $\frac{\sin(2n)+4}{n}$.
2. Determine whether the series $\sum_{n=1}^{\infty} n^2 e^{-n}$ converges or diverges.
3. Find the radius and the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+2)^n 3^{-n}}{n+1}$.
4. Use the first three non-zero terms of ta Maclaurin series to approximate the integral $\int_0^{0.2} \frac{\sin(x^2)}{x}\, dx$.
5. Evaluate the double integral $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)\, dydx$.

**Question 2 [3+3 Marks]**

a) Find the domain of the function $f(x,y) = \frac{\sqrt{x^2+y^2}}{x^2-4y^2}$ and find $\lim_{(x,y)\to(0,0)} f(x,y)$.

b) Find the area of the surface $x^2 + y^2 + z^2 = 36$ that lies above the plane $z = 3$.

**Question 3 [3+3+3 Marks]**

a) Use Chain rule to find $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$ where $w = r^2 + s^2, r = x^2 - y$ and $s = x + y^2$.

b) Find the points on the paraboloid $z = x^2 + y^2$ at which the normal line is parallel to the line through the points $A(2,0,0)$ and $B(1,2,1)$.

c) Find the directional derivative of $f(x,y,z) = xze^y + \cos(xy)$ at the point $P(1,0,2)$ in the direction of the line $x = 1 + 2t, y = -1 + t, z = 2 - 2t$. In which direction it increases most rapidly? What is the maximum rate of increase of $f$ at $P$?

**Question 4 [4+4 Marks]**

(a) Find local extrema and saddle points, if any, on the surface

$$f(x,y) = x^3 + y^2 - 3xy.$$

(b) Use method of Lagrange multiplier to find extrema of the function $f(x,y) = x - y$, subject to constraint $x^2 + 4y^2 = 4$.