

**Question 1** [4,4] a) Find the largest local interval for which the following initial value problem has a unique solution

$$\begin{cases} (x^2 - 9)y'' + y \sec x = e^x \\ y(2) = 1, y'(2) = 0. \end{cases}$$

b) By using the method of undetermined coefficients, find only the form of the particular solution of the differential equation

$$y^{(6)} - y'' = 6 + 5e^{-x} + 3x^2 \cos x + 7 \sin(10x)$$

**Question 2** [4,3]. a) Solve the initial value problem

$$\begin{cases} 5y'' - y' = e^x \\ y(0) = 0, y'(0) = 1 \end{cases}$$

b) Determine whether the functions

$$f_1(x) = \cos(2x - 1440), \quad f_2(x) = 4 \cos 2x, \quad f_3(x) = 2 \sin 2x,$$

are linearly independent or linearly dependent on  $(-\infty, \infty)$ .

**Question 3** [5] Find the general solution of the differential equation

$$xy'' + y' = 12 \ln x, \quad x > 0.$$

**Question 4** [5] Solve the following linear system of differential equations for  $x(t)$  only.

$$\begin{cases} x' + y' = 7x - 3y \\ -2x + y' = e^t \end{cases}$$