

**Question 1.** [4] Find the largest interval for which the following initial value problem has a unique solution:

$$\begin{cases} (\ln(x-2))y'' + \cos(x)y' + \sin(x)y = x \\ y(\pi) = 1, y'(\pi) = 0. \end{cases}$$

**Question 2.** [4] Show whether the functions

$f_1(x) = \sin(x)$ ,  $f_2(x) = \cos(x)$ , and  $f_3(x) = 1$   
are linearly independent or linearly dependent on  $\mathbb{R}$ .

**Question 3.** [4] Solve the nonhomogeneous differential equation:

$$y'' + y' - 2y = e^x.$$

**Question 4.** [4] Find the general solution of the differential equation:

$$y'' + y = \frac{1}{\sin(x)}, \quad x \in (0, \frac{\pi}{2}).$$

**Question 5.**[4] Solve the differential equation:

$$x^2y'' + 3xy' + 13y = 0, \quad x > 0.$$

**Question 6.**[5] Solve the following linear system:

$$(S) \quad \begin{cases} x' + y'' = e^t \\ x' + y' + x - y = e^t. \end{cases}$$