Question 1[4,4]. a) If $y = \sec x$ is a solution of the homogeneous differential equation.

$$y'' - (\tan x)y' - (\sec^2 x)y = 0,$$

then find the general solution of the nonhomogeneous equation

$$y'' - (\tan x)y' - (\sec^2 x)y = \sin x, \quad x > 0.$$

b) Solve the differential equation

$$4x^2 y'' + y = 0, \qquad x > 0,$$

subject to the initial conditions $y(1) = 2, y'(1) = 4$.

Question 2[4,4]. a) Obtain the general solution of the differential equation

$$y'' - y' - 6y = -e^x + 12x,$$

b) Find a linear homogeneous differential equation that has fundamental set of solutions $\{7, \cos 2x, 3x, 8\sin 2x, 5x^2, 2e^{-x}\}$.

Question 3[4,5]. a) Find an interval on which the Initial value problem

$$\begin{cases} (1 - 16x^2)y'' + \ln(5 - x^2)y' + \frac{y}{\sqrt{x}} = \sin^{-1}(2x) \\ y(\frac{3}{8}) = 0, \; y'(\frac{3}{8}) = 1. \end{cases}$$

admits a unique solution.

b) Solve the system of linear differential equations

$$\begin{cases} \frac{dx}{dt} + y - x = t \\ \frac{dy}{dt} - x - 3y = -3t \end{cases}$$