

**Question 1** [4,4] a) Find the largest local interval for which the following initial value problem has a unique solution

$$\begin{cases} (x-1)^3 y'' + y' \ln(3-x) + \frac{1}{\sqrt{x}} y = e^x \\ y(2) = 0, y'(2) = 1. \end{cases}$$

b) By using the method of undetermined coefficients, find only the form of the particular solution of the differential equation

$$y''' - y'' - 4y' + 4y = -3xe^x + 5e^{-2x} + \sin(2x).$$

**Question 2** [3,5]. a) Determine a homogeneous differential equation with constant coefficients having the set of fundamental solutions

$$\{2, e^{-x}, 3x, 5 \sin x\}.$$

b) If  $y_1 = e^{-2x}$  is a solution of the differential equation

$$(1+2x)y'' + 4xy' - 4y = 0, \quad x > -\frac{1}{2},$$

then find its general solution..

**Question 3** [5] Find the general solution of the differential equation

$$x^2 y'' - 2y = \frac{1}{x}, \quad x > 0.$$

**Question 4** [4] Show whether the functions  $f_1(x) = \sin x$ ,  $f_2(x) = \cos x$ ,  $f_3(x) = \sin(x-5)$  are linearly dependent or independent on  $\mathbb{R}$ .