جامعة الملك سعود
King Saud University

College of Science
Math Department

Final Exam M-204
Differential Equations
Duration : 3 hours

1439 - 1440 H (Summer semester)

Question 1 : (5)
Find and sketch the largest region of the xy-plane on which the initial value problem

$$y' = \sqrt{(2x+y)^2 - 4}$$

$$y(3) = 1$$

has a unique solution.

Question 2 (A) : (5)
Show that differential equation

$$\left(-2x + \frac{xy}{x+1} + y\ln(x+1)\right) dx + x\ln(x+1)\ dy = 0 \quad \text{for} \quad x > -1$$

is exact and then solve it.

Question 2 (B) : (5)
The population of a town grows at a rate proportional to the population at any time. Its population in year 2000 was 1 million, then 1.21 million in 2010.
a) What was the population in 2015 ?
b) What will be the population in 2020 ?

Question 3 (A) : (5)

a) Show that $y_1 = x^{-\frac{1}{2}} \cos x$ satisfies differential equation

$$(E) \quad 4x^2y'' + 4xy' + (4x^2 - 1)y = 0 \quad \text{for} \quad 0 < x < \frac{\pi}{2}.$$

b) Using a direct formula, find a second solution y_2 so that y_1 and y_2 are linearly independent on $(0, \frac{\pi}{2})$.
c) Deduce the general solution of (E).

Question 3 (B) : (5)
Solve differential equation

$$y'' - 4y' + 4y = 2\cosh(2x)$$

using undetermined coefficients method.

Question 4 (A) : (5)

Solve differential equation

$$y'' + y = \tan x \quad \text{for} \quad 0 < x < \frac{\pi}{2}$$

using variation of parameters method.

Question 4 (B) : (5)
Find the first seven terms in a power series expansion about ordinary point 0 for a solution to initial value problem

$$y'' + xy = 20 + 4x$$

$$y(0) = -32 \quad , \quad y'(0) = -48$$

Question 5 : (5)
Find the Fourier series of the function

$$f(x) = \pi - |x| \quad \text{for} \quad -\pi \le x \le \pi$$

$$f(x + 2\pi) = f(x) \text{ for all } x \ .$$

Deduce that

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \ .$$