King Saud University, Mathematics Department
Math 204. Time: 3H, Full Marks: 40, 17/08/2016
Final Exam (Summer Session)

Question 1. [4,4] a) Solve the initial value problem

$$\begin{cases} y' = \cos^2 y.\cos^2 x \\ y(0) = \frac{\pi}{4} \end{cases}$$

b) Find the general solution of the differential equation

$$[\sin(x+y) + y\cos(x+y) + x + y]\,y' + (y\cos(x+y) + y + x) = 0$$

Question 2. [4,5] a) Obtain the solution of the following initial value problem

$$y' = \frac{x+3y^2}{2y}, \quad y(0) = 1.$$

b) Write down the form of the particular solution of the differential equation

$$y''' + y' = 6 - 2\sin x + e^{-x}\cos 2x$$

Question 3. [4,4] a) Solve the differential equation

$$y'' - y = 5 + \cosh x.$$

b) Find the genral solution of the differential equation

$$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = \frac{1}{x+1}, \quad x > 0.$$

Question 4. [5] Solve the system of differential equations

$$\begin{cases} \frac{dx}{dt} + 2y + x = 0 \\ 2\frac{dx}{dt} + \frac{dy}{dt} + y = 0 \end{cases}$$

Question 5. [5,5] a) Let $f(x) = 1 - \frac{2x}{\pi}$, for all $x \in [0,\pi]$, such that $f(x+2\pi) = f(x)$. Sketch the graph of f on $[-2\pi, 2\pi]$ and find its Fourier series. Deduce that

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}.$$

b) Consider the function

$$g(x) = \begin{cases} 0, & x \le 0 \\ x, & 0 < x \le 1 \\ 0, & x > 1, \end{cases}$$

Skech the graph of f and find its Fourier integral. Deduce that

$$\int_0^{\infty} \frac{1-\cos\lambda}{\lambda^2}\,d\lambda = \frac{\pi}{2}.$$

(Hint take $x = 1$).