DEPARTMENT OF MATHEMATICS M - 204 TIME: 3 HOURS

KING SAUD UNIVERSITY FULLMARKS:100

(Summer semester, 1435-1436)

Question: 1 (a) Find the interval of xy- plane on which the initial-value problem

$$\frac{dy}{dx}=\frac{1}{x}\ln(1-x^2-y^2),\quad y(\frac{1}{2})=0. \qquad [5]$$

has a unique solution

(b) Solve the differential equation

$$\frac{dy}{dx}=\frac{y^2+xy+x^2}{x^2}. \qquad [10]$$

(c) Solve the initial-value problem

$$\frac{dy}{dx}=\frac{1}{y-x},\quad y(-1)=0. \qquad [10]$$

Question: 2 (a) Find the orthogonal trajectories for the family of curves $y=e^{2cx}$ **[10]**

(b) Find the differential equation whose general solution is

$$y=c_1e^x+e^{-x}(c_2\cos 3x+c_3\sin 3x) \qquad [5]$$

(c) Use reduction of order to solve the differential equation

$$x^2y''+xy'-y=x^2+1.$$

if $y_1=x$ **is given solution. [10]**

Question: 3. (a) Solve the non-homogenous differential

$$y''-3y'+2y=\cos(e^{-x}) \qquad [10]$$

(b) Find Power series solutions of non-homogenous differential equation

$$y''-xy'-y=0$$

about the ordinary point $x=0$. **[10]**

(c) Solve the system of differential equations

$$\frac{dx}{dt}=y+2t$$

$$\frac{dy}{dt}=1-t+2x-\frac{dx}{dt} \qquad [10]$$

Question: 4 (a) Expand $f(x)=e^x,\quad -\pi<x<\pi,\quad f(x+2\pi)=f(x),$

in a Fourier series. [10]

(b) Find half range Fourier series of the function

$$f(x)=x^2+2,\qquad 0<x<2 \qquad [10]$$