Question.1. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{1-x-y}{x+y}$$ [10]

(b) Verify that the differential equation

$$(-xy\sin x + 2y\cos x)dx + (2x\cos x)dy = 0.$$

is not exact and $\mu(x,y) = xy$ is the integrating factor of the equation.

Hence solve the differential equation.   [10]

Question. 2 (a) A radio active material has initial mass of 100 mg. After two years its mass is 75mg. Find the amount of the material at any time.

What is its half – life?   [10]

(b) Solve the second order linear differential equation

$$y'' - 2y' = \sin x + e^{2x}$$ [10]

(c) Solve the initial value problem

$$x^2 y'' + 3xy' + 3y = 0, \quad y(1) = 2, \quad y'(1) = 4, \quad x > 0.$$ [10]

Question . 3.(a) Solve the system of differential equations for ' y'

$$\frac{dx}{dt} + \frac{dy}{dt} = 2e^t - x + y$$

$$3x + 2y = \frac{dy}{dt} + t$$ [10]

(b) Find the general solution for the differential equation

$$y'' + xy' = -y \text{ about the ordinary point } x = 0.$$ [10]

Question . 4. (a) Is the set of functions $\{\sin nx\}, n = 1,2,3,....$ orthogonal on the interval $[-\pi, \pi]$? Find the norm of each function.   [10]

(b) Find the Fourier series of the function

$$f(x) = \pi - x, \quad if -\pi < x < \pi$$

$$f(x + 2\pi) = f(x)$$ [10]

(c) Find the Fourier integral representation of the function   [10]

$$f(x) = \begin{cases} 0 & , if \quad x < -1 \\ x^2 + x & , if \quad -1 \le x \le 1 \\ 0 & , if \quad x > 1 \end{cases}$$