

**King Saud University Department of Mathematics**  
**FINAL EXAM (MATH 204). TIME: 3H, FULL MARKS: 40**

**Question 1.** [5,4] a) Initially, there were 100 milligrams (mg) of a radioactive substance. After 6 hours, the mass decreased by 3%. Assuming the rate of decay is proportional to the amount remaining, determine the half-life of the substance.

b) Solve the differential equation

$$(y + xy + 1)dy + y^2 dx = 0, \text{ for } y > 0.$$

**Question 2.** [4,5] a) Solve the initial value problem

$$e^{x-y} \ln x dx - (\ln xy - \ln y) dy = 0, \quad x > 1, \quad y(2) = 2.$$

b) Use power series method to solve the initial value problem

$$y'' + xy = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

**Question 3.** [4,4,4] a) Find the largest interval for which the following initial value problem admits a unique solution

$$\ln(5 - x^2)y'' + (\sin x)y = x^{-1}, \quad y(-1) = 1, \quad y'(-1) = 0.$$

b) Solve the differential equation:

$$y'' + y = \frac{1}{1 + \sin x}.$$

c) Find the general solution of the differential equation

$$4x^2 y'' + 8xy' + y = \sqrt{x}.$$

**Question 4.** [5,5] a) Find the Fourier sine series of the function

$$f(x) = \begin{cases} 1 & \text{if } 0 < x < \pi \\ 0 & \text{if } x = \pi \end{cases}$$

Deduce the value of the numerical series  $\sum_{n=0}^{\infty} \frac{(-1)^k}{2k+1}$ .

b) Compute the Fourier integral of the function:  $f(x) = \begin{cases} \cos x, & |x| < \pi \\ 0, & |x| \geq \pi \end{cases}$

Deduce the value of the integral  $\int_0^{\infty} \frac{\lambda \sin(\lambda \pi)}{1 - \lambda^2} d\lambda$ . (Hint:  $2 \cos a \cos b = \cos(a+b)$

$b) + \cos(a - b)$ ).