KING SAUD UNIVERSITY DEPARTMENT OF MATHEMATICS
M204.TIME 3H, FULL MARKS 40, FINAL EXAM S2-2025/26

Question 1. [4,4,5] a) Solve the initial value problem

$$\begin{cases} \frac{dy}{dx} = 2xy - x - 2y + 1 \\ y(0) = 1 \end{cases}$$

b) Obtain the general solution of the differential equation

$$\left(x\sin(\frac{y}{x}) - y\cos(\frac{y}{x})\right)dx + x\cos(\frac{y}{x})dy = 0$$

c) A small metal bar, whose initial temperature was 20^0C, is dropped into a large container of boiling water. How long will it take the bar to reach 90^0C if its temperature increases at a rate of 2^0C per second at the moment it was dropped? How long will it take the bar to reach 98^0C (Hint: boiling water100^0C)

Question 2. [4,4] a) Solve the differential equation

$$(\cos x \sin x)y' + y = \cos^2 x \sin x, \quad x \in (0, \frac{\pi}{2})$$

b) Find the general solution of the differential equation

$$(x+5)y'' + 5y' + \frac{3}{x+5}y = 1, \; x \neq -5$$

Question 3. [4,5] a) Write down only the form of the particular solution y_p of the differential equation

$$y^{(5)} + 4y^{(3)} = 4x + \frac{x}{3}e^{-x} + 5\cos(2x)$$

b) Find the power series solution about the ordinary point $x_0 = 0$ for the differential equation

$$y'' + (2x-1)y' - y = x.$$

Question 4. [5,5] a) Find the Fourier series for the 2π-periodic function defined by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } -\pi < x \leq 0 \end{cases}$$

Sketch the graph of f on the interval $(-3\pi, 3\pi)$. Deduce the value of the numerical series $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$ and $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.

b) Consider the function f defined by

$$f(x) = \begin{cases} \sin x, & |x| \leq \pi \\ 0, & |x| \geq \pi \end{cases}$$

Sketch the graph of f. Obtain its Fourier integral, and deduce the value of the integral $\int_0^{\infty} \frac{\sin^2 \lambda}{\pi^2 - \lambda^2} d\lambda$.

الممسوحة ضوئيا بـ CamScanner