King Saud University
Department Of Mathematics.
M-203 [Final Examination]
(Differential and Integral Calculus)
(I-Semester 1441)

Max. Marks: 40 Time: 3 hrs

Marking Scheme: Q1[4+4+4]; Q2[4+4+4]; Q3[4+4+4+4].

Q. No: 1 (a) Determine whether the sequence $\left\{\left(\frac{n+2}{n+3}\right)^n\right\}$ converges or diverges and if it converges, find its limit.

(b) Find the interval of convergence and radius of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n}.$$

(c) Find the MacLaurin series for $f(x) = \cos^2(x)$ and use its first three non-zero terms to approximate the integral $\int_0^1 \cos^2(\sqrt{x})\ dx.$

Q. No: 2 (a) Evaluate the integral

$$\int_0^{\sqrt{\frac{\pi}{2}}} \int_y^{\sqrt{\frac{\pi}{2}}} \cos(x^2)\, dxdy.$$

(b) Find the moment of inertia about the x-axis of the lamina with shape of the region bounded by $y = x^2$ and $y = 0,$ and $x = 1$ with density $\delta = x + y.$

(c) Evaluate the triple integral:

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} z\sqrt{x^2+y^2}\, dzdydx.$$

Q. No: 3 (a) Show that the following integral is independent of path and find its value:

$$\int_{(0,1)}^{(1,2)} (y + 2xy)dx + (x^2 + x)dy.$$

(b) Use Green's theorem to evaluate the line integral $\oint_C (e^x + x^3)\, dx + (yx^2 + y^3)dy$, where C is the path from $(0,0)$ to $(1,2)$ along the graph $y = 2x^2$ and from $(1,2)$ to $(0,0)$ along the graph $y = 2x$.

(c) Use Divergence theorem to evaluate the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dS,$ where $\vec{F}(x,y,z) = (x^2 + \sin(yz))\,\vec{\imath} + (\cos(xz) - 2xy)\vec{\jmath} + (e^y + z^2)\vec{k}$ and S is the surface of the region bounded by the cylinder $x^2 + y^2 = 1$, the *xy*-plane, and the paraboloid $z = 2 - x^2 - y^2$. (Provided S is oriented by the unit normal directed upward).

(d) Use Stoke's theorem to evaluate $\oint_C \vec{F} \cdot \overrightarrow{dr}$ for the vector field $\vec{F}(x,y,z) = 2z\vec{\imath} + 3x\vec{\jmath} + y\vec{k}$, S is the surface of the paraboloid $z = 1 - x^2 - y^2$ and C is the trace of S in the *xy*-plane with counterclockwise direction.