بسم الله الرحمن الرحيم

طريقة لوبي دالنهاية : تعريف النهاية - التعويض - لوبيتال - الساندويتش

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$0 \leftarrow a_n \Leftarrow 0 \leftarrow |a_n|$ — $r^n \to 0$ if $|r|<1$, $\to \infty$ if $|r|>1$

8.1 Sequences:

Determine whether the sequence converges or diverges, and if it converges, find the limit

(24) $\left\{\frac{n^2}{\ln(n+1)}\right\}$ (26) $\left\{\frac{\cos n}{n}\right\}$ (27) $\left\{\frac{e^n}{n^4}\right\}$

(38) $\left\{\frac{n^2}{2^n}\right\}$ (30) $\left\{(-1)^n n^3 3^{-n}\right\}$ (42) $\left\{\sqrt{n^2+n}-n\right\}$

8.2 convergent or divergent series:

geometric series $\sum ar^n = \frac{a}{1-r}$ is D if $|r| \geq 1$

$S_n \to D \Leftrightarrow \sum D$, $S_n \to C \Leftrightarrow \sum C$

(43) $\sum_{n=1}^{\infty}\left(2^{-n}-2^{-3n}\right)$ (45) $\sum_{n=1}^{\infty}\left[\frac{1}{8^n}+\frac{1}{n(n+1)}\right]$

(34) $\sum \frac{1}{1+(0.3)^n}$ (46) $\sum_{1}^{\infty}\left[\frac{1}{n(n+1)}-\frac{4}{n}\right]$

كثير الحدود البسيطة ... $\lim = \infty$

8.3 Positive-term Series [Integral, P-series, B-comparison, limit comparison]

$\int_1^\infty f(x)\,dx$ $p>1$ C, $p \leq 1$ D

Determine whether the series converges or diverges

integral $\leftarrow$ (9) $\sum_{n=2}^{\infty}\frac{1}{n\sqrt{n^2-1}}$ (15) $\sum_{n=1}^{\infty}\frac{1}{n3^n}$ (23) $\sum\frac{1}{\sqrt{4n^3-5n}}$ $\sum\frac{1}{n^{3/2}}$

(27) $\sum\frac{1}{\sqrt{n}+9}$ $\sum\frac{1}{\sqrt{n}}$ (31) C $\sum\frac{1+2^n}{1+3^n}$ (32) $\sum\frac{3n}{2n^2-7}$

(33) D $\sum\frac{1}{\sqrt[3]{5n^2+1}}$ (42) D $\sum\frac{n+\ln n}{n^2+1}$ $\sum\frac{1}{n} \leq \sum\frac{n}{n^2+1}$ (45) C $\sum\frac{\ln n}{n^3}$

8.4 The Ratio and Test tests ($a_n \geq 0$)

(36) $\sum\frac{(2n)!}{2^n}$ (2) $\sum_{n=1}^{\infty}\frac{n^n}{10^{n+1}}$

not alternating both terms [≥ 0, ≤ 0]

8.5 Alternating series and absolute convergence

A.S.T : $\sum(-1)^n a_n$, $a_n>0$, $a_n \downarrow$, ...

$\sum$ is D $\Uparrow$ (1) $\sum(-1)^{n-1}\frac{1}{n^2+7}$ $C \Leftarrow r<1$

D $\Leftarrow$ ... or $\lim\left|\frac{a_{n+1}}{a_n}\right|>1$

AC, CC, D : (9) D $\sum(-1)^n \frac{n}{\ln n}$ AC (19) $\sum_{n=1}^{\infty}\frac{\cos\frac{1}{6}\pi n}{n^2}$

AC: $\sum|a_n|$ C; CC: $\sum|a_n|$ D, $\sum a_n$ C

$\sum a_n$ or $\lim\left|\frac{a_{n+1}}{a_n}\right|$

Limit Comparison Test: $a_n, b_n \geq 0$

(i) If $\lim\frac{a_n}{b_n}=0$ and $\sum b_n$ C $\Rightarrow \sum a_n$ C

(ii) If $\lim\frac{a_n}{b_n}=\infty$ and $\sum b_n$ D $\Rightarrow \sum a_n$ D

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8.6 Power Series

Find the interval of convergence of the power series

(21) $\sum \frac{n^2}{2^{3n}}(x+4)^n$ $(-12, 4)$ (29) $\sum (-1)^n \frac{3^n}{n!}(x-4)^n$ $(-\infty, \infty)$

8.7 Power series representations of functions

Find the power series in x that has the given sum and specify the radius of convergence $\frac{1}{1-x} = \sum x^n$ $\frac{1}{1+x} = \sum (-1)^n x^n$

(5) $\frac{x^2}{1-x^2}$ (18) $f(x) = x e^{-3x}$ (19) $f(x) = x^2 \ln(1+x^2)$, $|x|<1$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\ln(1+x^2) = \int_0^x \frac{2t}{1+t^2}\,dt$

(23) $f(x) = \sinh(-5x)$ $\sum \frac{-5^{2n+1}}{(2n+1)!} x^{2n+1}$

(34) $f(x) = \ln(3+2x)$

Use infinite series to approximate the integral to four decimal places $\sum (-1)^n a_n$

(27) $\int_0^{\frac{1}{3}} \frac{dx}{1+x^6}$ $\frac{\frac{1}{3}}{1} - \frac{(\frac{1}{3})^7}{7} + \frac{(\frac{1}{3})^7}{13}$ $|S - S_1| \le a_2 = \frac{(\frac{1}{3})^7}{7} = 0.000065$

≈ 0.333

8.8 Maclaurin and Taylor Series

Find ~~Use~~ Maclaurin series for $\cos x = \sum (-1)^n \frac{x^{2n}}{(2n)!}$ $(-\infty, \infty)$

(13) $f(x) = \cos^2 x = \left(\frac{\cos(2x)+1}{2}\right)$

Find a Taylor series for $\sum \frac{f^{(n)}(c)}{n!}(x-c)^n$

(19) $f(x) = \frac{1}{x}$ at $c = 2$.

Find the first three terms of a Taylor series for:

(23) $f(x) = \sec x$ at $c = \frac{\pi}{3}$

Use the first two nonzero terms of a Maclaurin series to approximate the number (33) $\tan^{-1}(0.1)$, and estimate the error in the approximation

(36) $\int_0^{\frac{1}{2}} x\cos(x^2)\,dx$ $\tan^{-1}x = \sum (-1)^n \frac{x^{2n+1}}{2n+1}$ $[-1,1]$

Approximate the improper integral to four decimal places (40) $\int_0^1 \frac{\sin x}{x}\,dx$

0.9461 , 1.0516 $\sin x = \sum (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $(-\infty, \infty)$

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13.3 Double Integrals in Polar coordinates

x (12) Use double integral to find the area of ~~one loop of~~ inside $r = 3\sin\theta$ outside $r = 1 + \sin\theta$ ~~$r^2 = 9\cos(2\theta)$~~

Use Polar coordinates to evaluate the integral

(21) $\int_1^2 \int_0^x \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx$ $\quad \ln(\sqrt{2}+1)$

(23) $\int_0^2 \int_0^{\sqrt{4-y^2}} \cos(x^2+y^2)\, dx\, dy$ $\quad \frac{\pi}{4}\sin 4$

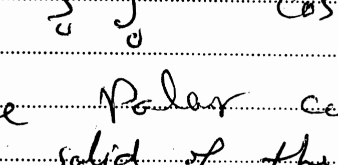
Use Polar coordinates to find the volume of the solid of the ~~shape~~ Q bounded by the cone $z^2 = x^2+y^2$ and the cylinder $x^2+y^2 = 2x$

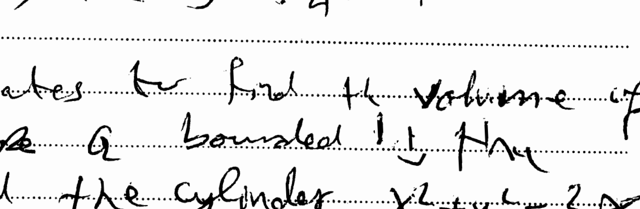
$\iint f(x,y)\, r\,dr\,d\theta = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r\,dr\,d\theta = \frac{64}{9}$

13.4 Surface Area

(11) Find the surface area of the part of the sphere $x^2+y^2+z^2 = a^2$ that is inside the cylinder $x^2+y^2 = ay$ $\quad 2a^2(\pi-2)$

(8) Find the surface area of the first octant portion of the cylinder $y^2+z^2=9$ that lies inside the cylinder $x^2+y^2=9$





13.5

Sketch and find its volume

$\frac{128}{3}$ (12) $x^2+z^2=4,\ y^2+z^2=4$ $\quad V = 8\left[\int_0^2\int_0^y\int_0^{\sqrt{4-y^2}} dz\,dx\,dy + \int_0^2\int_0^x\int_0^{\sqrt{4-x^2}} dz\,dy\,dx\right]$

2π (15) $y^2+z^2=1,\ x+y+z=2,\ x=0$ $\quad V = \int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_0^{2-y-z} 1\, dx\,dz\,dy$

$\frac{81\pi}{32}$ (16) $z = x^2+y^2,\ y+z=2$ $\quad V = \int_{-2}^{1}\int_{y^2}^{2-y}\int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} 1\, dx\,dz\,dy$

(26) A lamina having area mass density $\delta(x,y) = x^2+y^2$ has the shape of the region bounded by. Set up integral ~~wh~~ can be used to find the mass of lamina

$xy^2 = 1,\ x=0,\ y=1,\ y=2$ $\quad m = \iint_R \delta\, dA$

(31) A solid having density $\delta(x,y,z) = x^2+y^2$, has the shape of ~~Sup~~ Set up ... mass of the solid $\quad m = \iiint_Q \delta\, dV$

$x+2y+z=4,\ x=0,\ y=0,\ z=0$

13.6

(4) Find the mass and center of mass of lamina that has the shape — and has area mass density at $P(x,y)$ is directly proportional to the distance from x-axis to P $\delta = ky$

$y = x^3$, $y = 2x$

$m = \iint_R \delta(x,y)\,dA$

$M_x = \iint_R y\,\delta(x,y)\,dA$

$M_y = \iint_R x\,\delta(x,y)\,dA$

$\bar{x} = \frac{M_y}{m}$, $\bar{y} = \frac{M_x}{m}$

(12) Find I_x, I_y and I_0 (Ex 4)

$I_x = \iint_R y^2 \delta\, dA$

$I_y = \iint_R x^2 \delta\, dA$, $I_0 = \iint_R (x^2+y^2)\delta\, dA$

(20) Set up the center of mass of the solid

$\delta(x,y,z) = x^2y^2z^2$, Q bounded by ~~paraboloid~~ $\cancel{x = y^2+z^2}$ hyperboloid $y^2 - x^2 - z^2 = 1$ and plane $y = 2$

$\bar{x} = \frac{M_{yz}}{m}$; $\bar{y} = \frac{M_{xz}}{m}$; $\bar{z} = \frac{M_{xy}}{m}$

$M_{yz} = \iiint_Q x\,\delta\, dV$, ...

(24) Set up the iterated integrals can be used to find the centroid of the solid bounded by the graphs of the equations $z = x^2$, $y = x^2$, $y = x^3$, and $z = 0$

(centroid $= (\bar{x}, \bar{y}, \bar{z})$)

$\delta(x,y,z) = k$

(25) Set up an iterated integral that can be used to find the moment of inertia with respect to the z-axis of the solid:

The sphere of radius a with center at the origin

$\delta(x,y,z) = x^2+y^2+z^2$

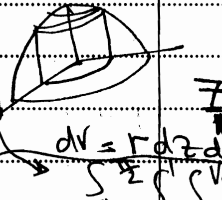
$I_z = \iiint_Q (x^2+y^2)\,\delta\, dV$

13.7 Cylindrical Coordinates

(32) A solid is cut out of the sphere $x^2+y^2+z^2 = 4$ by the cylinder $x^2+y^2 = 2y$. The density at $P(x,y,z)$ is directly proportional to the distance from xy-plane to P. Find (i) its mass (ii) its center of mass $\delta = kz$

(39) Use cylindrical coordinates to ev

$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{4-x^2-y^2}} z\, dz\, dx\, dy$



$\frac{7\pi}{16}$

$dV = r\,dz\,dr\,d\theta$

$\int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{\sqrt{4-r^2}} z\,r\,dz\,dr\,d\theta$

13.8 Spherical Coordinates

(2) $(1, \frac{3\pi}{4}, \frac{2\pi}{3}) \to (x,y,z)$ $\to (r,\theta,z)$

ρ, ϕ, θ

$z = \rho\cos\phi$

(8) $(1, 1, -2\sqrt{3}) \to (\rho, \phi, \theta)$ $\to (z, r, \theta)$

(37) The density at $P(x,y,z)$ is directly proportional to square of the distance from the center of the sphere to P

$\delta = k(x^2+y^2+z^2)$

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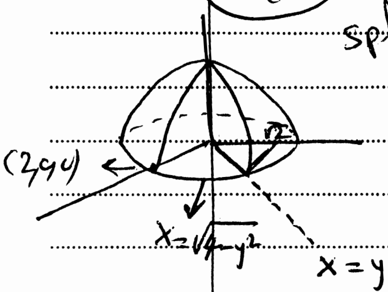
(40) Evaluate the integral by changing to spherical coordinates.



$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dx\, dy$$

$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^{2} (\rho)\, \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta = \pi$$

14.1

(13) Find a conservative vector field that has the given potential $f(x,y,z) = x^2 - 3y^2 + 4z^2$

$F = \nabla f = f_x\, i + f_y\, j + f_z\, k$

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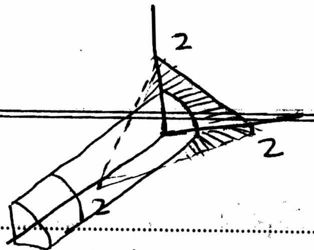

13.5 Triple Integrals.

(15) Sketch the region bounded by the graphs of the equation, and use a triple integral to find its volume

2π

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{0}^{2-y-z} dx\,dz\,dy$$

$y^2+z^2=1$, $x+y+z=2$, $x=0$

(30) A lamina having area mass density $\delta(x,y)$ has the shape of the region bounded by the graphs of the equations. Set up an iterated double integral that can be used to find the mass of the lamina

$m=\iint_R \delta(x,y)\,dA$

$\delta(x,y)=x^2+y^2$, $xy^2=1$, $x=0$, $y=1$, $y=2$

13.6 Moments and center of mass.

(4) Find the mass and center of mass of the lamina that has the shape of the region bounded by the graphs of the given equations and has the area mass density ~~$\delta(x,y)$~~

$y=x^3$, $y=2x$

density at $P(x,y)$ is directly proportional to the distance from the x-axis to P $\rightarrow \boxed{\delta(x,y)=ky}$

$m=\iint_R \delta(x,y)\,dA$, $M_x=\iint_R y\,\delta(x,y)\,dA$, $M_y=\iint x\,\delta(x,y)\,dA$

$\bar{x}=\frac{M_y}{m}$, $\bar{y}=\frac{M_x}{m}$

(12) Find $I_x=\iint_R y^2\delta(x,y)\,dA$, $I_y=\iint_R x^2\delta(x,y)\,dA$

$I_0=\iint_R (x^2+y^2)\,\delta(x,y)\,dA$ (OF EX. 4)

(20) Set up iterated integrals that can be used to find the center of mass of the solid that has the shape of the region G and the density $\delta(x,y,z)$

$\bar{x}^2\ y^2\ z^2$ 1, G is bounded by the hyperboloid $y^2-x^2-z^2=1$ and the plane $y=2$,

$\bar{x}=\frac{M_{yz}}{m}$, $\bar{y}=\frac{M_{xz}}{m}$, $\bar{z}=\frac{M_{xy}}{m}$

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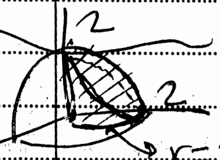
(25) Set up an iterated integral that can be used to find the moment of inertia with respect to z-axis of the sphere of radius a with center at the origin; $\delta(x,y,z) = x^2+y^2+z^2$

$$I_z = \iiint_Q (x^2+y^2)\,\delta(x,y,z)\,dV$$



13.7 cylindrical

(38) A solid is cut out of the sphere $x^2+y^2+z^2=4$ by the cylinder $x^2+y^2=2y$. The density at $P(x,y,z)$ is directly proportional to the distance ~~from the~~ xy-plane to P. Find:

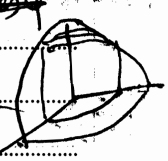
$\delta = kz$

$\frac{\pi}{2}k$ (a) its mass   (b) its center of mass $\cancel{(0,0,\frac{2a}{5})}$ $(0,\frac{4}{5},0)$

(39) Use cylindrical coordinates to evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{4-x^2-y^2}} z\,dz\,dx\,dy$$

$\cancel{z=\rho\cos\phi}$



13.8 spherical coordinates:

(37) Find the mass of solid that lies outside the sphere $x^2+y^2+z^2=1$ and inside the sphere $x^2+y^2+z^2=4$ if the density at a point P is directly proportional to the square of the distance from the center of the sphere to P.

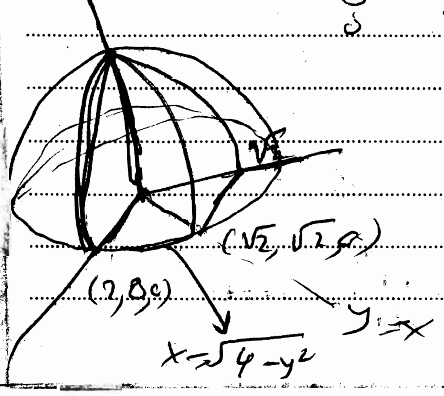
$\frac{124\pi k}{5}$

(40) Evaluate the integral by changing to spherical coordinates

$$\int_0^{\sqrt{2}} \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz\,dx\,dy$$

$z = \rho\cos\phi$

π



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Work $= \int_C F \cdot dr$ — dot — $xi + yj + zk = r \Rightarrow dr = dx\,i + dy\,j + dz\,k$

(24) $F = zi + xj + yk$ W=? along twisted cubic $x = t, y = t^2, z = t^3$ from $(0,0,0)$ to $(2,4,8)$ — $\frac{412}{15}$

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14.2 Line integrals

(5) Evaluate the integral along C

$\int 6x^2\,dx + xy\,dy$; C is the graph of $y = x^3+1$ from $(-1,0)$ to $(1,2)$. — $\frac{34}{7}$

$x = t \to dx = dt$
$y = t^3+1 \to dy = 3t^2 dt$
$-1 \le t \le 1$
$\int_{-1}^{1} \dots dt$

14.3

~~(5) Show that $\int_C F \cdot dr$ is independent of path by finding a potential f for F.~~

$F = f_x i + f_y j$
$\Rightarrow f(x,y) = \dots$ potential function

(11) Show that the line integral is independent of path, and find its value

$$\int_{(-1,2)}^{(3,1)} (y^2+2xy)\,dx + (x^2+2xy)\,dy$$

$\int F \cdot dr = f(x,y)\Big]_{(-1,2)}^{(3,1)} = \dots = 14$

نأخذ C=0 لأن (ما تفرق)

$f_x = y^2 + 2xy$
$\Rightarrow f = y^2x + x^2y + g(y)$
$x^2 + 2xy = \frac{\partial f}{\partial y} = 2yx + x^2 + g'(y)$
$\Rightarrow g(y) = c$
$\boxed{f = y^2x + x^2y + c}$

(15) Use Th(14.16) to show that $\int_C F \cdot dr$ is not independent of Path

$\Longleftrightarrow \quad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$F(x,y) = \underbrace{4xy^3}_{M\text{ cts and derivative}} i + \underbrace{2xy^3}_{N\text{ cts and derivative}} j$

?

نفس طلوب (10) $F(x,y,z) = 2x\sin z\, i + 2y\cos z\, j + (x^2\cos z - y^2\sin z)\,k$

(11) $F(x,y,z) = \dots + g(y,z)$

التعويض بشكل $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$, $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$

14.4 Green's Theorem:

Use Green Theorem to evaluate the line integral.

$$\oint_C M\,dx + N\,dy = \iint \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$\frac{2}{3}$ (3) $\oint_C x^2y^2\,dx + (x^2-y^2)\,dy$; C is the square with vertices $(0,0), (1,0), (1,1), (0,1)$

π (5) $\oint_C xy\,dx + (y+x)\,dy$; " " circle $x^2+y^2=1$ by Polar

$\frac{128}{3}$ (15) Find area of region bounded by: $y = 4x^2$, $y = 16x$

(20) Find area " " bounded by C: $x = \frac{3t}{t^3+1}$, $y = \frac{3t^2}{t^3+1}$, $0 \le t \le 1$ — $A = \oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C (x\,dy - y\,dx)$

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14.6 Divergence Theorem:

(12) Verify D. Theorem $F = x^2 i + y^2 j + z^2 k$, S by $x = a$, $y = a$, $z = a$, $a > 0$ — the cube bounded by the coordinates planes and the planes

(2) Use the divergence theorem to find $\iint_S F \cdot n \, dS = \iiint_Q \nabla \cdot F \, dV$

$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$

$F = y^3 e^z i - xy j + x \tan^{-1}(y) k$;

S the surface of the region bounded by coordinate planes and the plane $x + y + z = 1$

(10) Use divergence Theorem to the flux of F through S $\iint_S F \cdot n \, dS$

$F(x, y, z) = x^3 i + y^3 j + z^3 k$

S is the surface of the region that inside both the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 25$

14.7 Stokes's Theorem:

$\oint_C F \cdot T \, ds = \iint_S (\text{curl} F) \cdot n \, dS$ — in xy plane of Q

(1) Verify Stokes's Theorem for F and S

$F = y^2 i + z^2 j + x^2 K$

S is the first-octant portion of the plane $x + y + z = 1$

$z = 1 - x - y = f(x, y)$

$\nabla \times F = \text{curl} F = \begin{vmatrix} i & j & K \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a & b & c \end{vmatrix}$

(5) If $F = (3z - \sin x) i + (x^2 + e^y) j + (y^3 - \cos z) k$

Use Stokes's theorem to evaluate $\oint_C F \cdot dr$, where C is the curve given by

$x = \cos t$, $y = \sin t$, $z = 1$, $0 \le t \le 2\pi$

$n = \frac{\nabla g(x, y, z)}{\| \nabla g(x, y, z) \|} = \frac{-f_x i - f_y j + k}{\sqrt{f_x^2 + f_y^2 + 1}}$

$g(x, y, z) = z - f(x, y)$ — the surface S

(7) If $F = 2y i + e^z j - \tan^{-1} x \, K$ and S the portion of the paraboloid $z = 4 - x^2 - y^2$ cut off by the xy-plane, use Stokes's theorem to evaluate $\iint_S \text{curl} F \cdot n \, dS$

$\iint_S f(x, y, z) \, ds = \iint_{R_{xy}} f(x, y, f(x, y)) \sqrt{f_x^2 + f_y^2 + 1} \, dA$

$\oint_C F \cdot T \, ds = \oint_C F \cdot dr$, $dr = (dx, dy, dz)$

هنا x, y تتحرك $z = 0$ ثابتة $dz = 0$

$A = \oint_C x \, dy = -\oint_C y \, dx = \frac{1}{2} \oint_C x \, dy - y \, dx$

$\frac{\partial p}{\partial y}$

$x \, dy =$

$x =$

$y - y \, dx$