King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second-Mid Term Examination
(First Semester 1437/1438)

Max. Marks: 25 Time: 90 minutes

Marking Scheme: All questions carry equal marks.

Q. No: 1 Evaluate the integral $\int_{0}^{1} \int_{\sqrt{y}}^{1} \sqrt{x^3+1}\ dxdy$.

Q. No: 2 Evaluate the integral $\iint_R y\ dA$, where R is the region in the first quadrant bounded by the circle $x^2+y^2=9$, the line $y=x$ and $y=0$.

Q. No: 3 Find the surface area of the part of the paraboloid $z=x^2+y^2$ that lies inside the cylinder $x^2+y^2=1$.

Q. No: 4 Find the centre of mass of the solid bounded by the region that lies inside the sphere $x^2+y^2+z^2=4z$ and below the cone $z=\sqrt{3x^2+3y^2}$ having density $\delta(x,y,z)=\dfrac{1}{x^2+y^2+z^2}$.

Q. No: 5 Find the volume of the solid bounded by the paraboloids $z=4-x^2-y^2$, $z=x^2+y^2$, and the cylinder $x^2+y^2=1$.

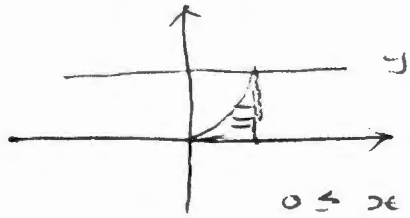
M-203 (1)

II Mid-term Exam (I sem. 1437/1438)

Q. No. 1 $\displaystyle\int_0^1\int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy$

$0 \le y \le 1$

$\sqrt{y} \le x \le 1$ [Marks: 5



$0 \le x \le 1$

$0 \le y \le x^2$

$$= \int_0^1\int_0^{x^2} \sqrt{x^3+1}\,dy\,dx \quad (3)$$

$$= \int_0^1 (x^3+1)^{1/2}\,x^2\,dx$$

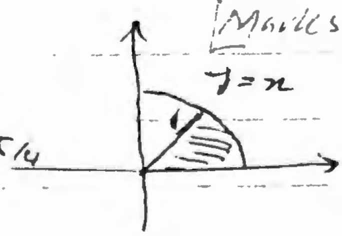
$$= \frac{1}{3}\int_0^1 (x^3+1)^{1/2}(3x^2)\,dx = \frac{1}{3}\,\frac{(x^3+1)^{3/2}}{3/2}\Big]_0^1$$

$$= \frac{2}{9}\left[(2)^{3/2}-1\right] \quad \# \quad (2)$$

Q. No. 2 $\displaystyle\iint_R y\,dA$ [Marks: 5]

$0 \le r \le 1$

$0 \le \theta \le \pi/4$



$$= \int_0^{\pi/4}\int_0^1 r^2 \sin\theta\,dr\,d\theta \quad (3)$$

$$= \int_0^{\pi/4} \frac{r^3}{3}\Big]_0^1 \sin\theta\,d\theta = \frac{1}{3}\left[-\cos\theta\right]_0^{\pi/4} = \frac{1}{3}\left[-\frac{1}{\sqrt{2}}+1\right] \quad (2)$$

Q. No. 3 $\displaystyle SA = \iint_R \sqrt{1+4x^2+4y^2}\,dA = \int_0^{2\pi}\int_0^1 \sqrt{1+4r^2}\;r\,dr\,d\theta \quad (3)$ [Marks: 5]

$$= \frac{1}{8}\int_0^{2\pi} \frac{(1+4r^2)^{3/2}}{3/2}\Big]_0^1 d\theta = \frac{1}{12}\left[(5)^{3/2}-1\right]2\pi$$

$$= \frac{\pi}{6}\left[5^{3/2}-1\right] \quad \# \quad (2)$$

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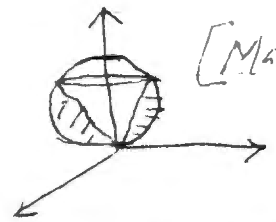
$\rho^2 = 4z$ (2)

Q: No. 4 mass $= \iiint_Q \frac{1}{x^2+y^2+z^2}\, dv$



[Marks: 5]

$$= \int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{4\cos\varphi} \frac{1}{\rho^2}\,\rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta \qquad (3)$$

$0 \le \rho \le 4\cos\varphi$

$\pi/6 \le \varphi \le \pi/2$

$0 \le \theta \le 2\pi$

$$= \int_0^{2\pi}\int_{\pi/6}^{\pi/2} \left[\rho\right]_0^{4\cos\varphi} \sin\varphi \, d\varphi\, d\theta$$

$$= \int_0^{2\pi}\int_{\pi/6}^{\pi/2} 4\sin\varphi\cos\varphi \, d\varphi\, d\theta = 2\int_0^{2\pi} \left[-\frac{1}{2}\cos 2\varphi\right]_{\pi/6}^{\pi/2} d\theta$$

$$= -\left[\cos\pi - \cos\pi/3\right] 2\pi = -\left[-1 - \frac{1}{2}\right] 2\pi$$

$$= 3\pi \qquad (1)$$

$$= \cancel{\left(1+\frac{\sqrt{3}}{2}\right) 2\pi}$$

$$\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{4\cos\varphi} \frac{1}{\rho^2}\,\rho\cos\varphi\,\rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{2\pi}\int_{\pi/6}^{\pi/2} \left[\frac{\rho^2}{2}\right]_0^{4\cos\varphi} \cos\varphi \sin\varphi \, d\varphi\, d\theta = 8\int_0^{2\pi}\int_{\pi/6}^{\pi/2} \cos^3\varphi \sin\varphi \, d\varphi\, d\theta =$$

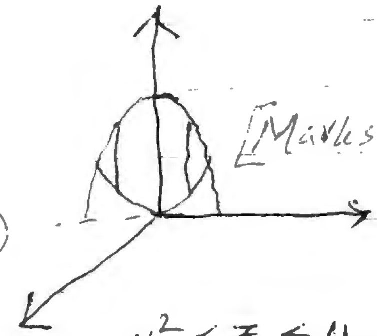
$$= -8\int_0^{2\pi} \left[\frac{\cos^4\varphi}{4}\right]_{\pi/6}^{\pi/2} d\theta = -8\left[0 - \frac{(\sqrt{3}/2)^4}{4}\right] 2\pi$$

$$= 2\left(\frac{9}{16}\right) 2\pi = \frac{9}{4}\pi$$

$\bar{x} = 0,\ \bar{y} = 0,\ \bar{z} = \dfrac{\frac{9}{4}\pi}{\left(1+\cancel{\frac{\sqrt{3}}{2}}\right) 2\pi}$

$\bar{z} = \dfrac{9/4\,\pi}{3\pi} = 3/4$

(3)



[Marks. 5]

$r^2 \le z \le 4 - r^2$

$0 \le r \le 1$

$0 \le \theta \le 2\pi$

Q:N5

$$V = \int_0^{2\pi}\int_0^1\int_{r^2}^{4-r^2} r \, dz \, dr \, d\theta \quad (3)$$

$$= \int_0^{2\pi}\int_0^1 r\left[4 - r^2 - r^2\right] dr \, d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left[4r - 2r^3\right] dr \, d\theta = \int_0^{2\pi}\left[\frac{4r^2}{2} - \frac{2}{4}r^4\right]_0^1 d\theta$$

$$= \left(2 - \frac{1}{2}\right) 2\pi = 3\pi \quad (2)$$