Department of Mathematics, College of Science, King Saud University

M-203 (Differential and Integral Calculus), II-Mid-term Examination,

Semester-I, 1446, 2024/2025

Time-90 Minutes. Max. Marks-25

All questions carry equal marks.

Q.1. Evaluate the double integral

$$\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{1}{1+x^4}\,dx\,dy.$$

Q.2. Evaluate the double integral

$$\iint_R \left(4+\sqrt{x^2+y^2}\right) dA,$$

where R is the region inside of the circle $x^2+y^2=2x$.

Q.3. Find the surface area of the portion of the cone $z=\sqrt{x^2+y^2}$ that lies inside the paraboloid $2z=x^2+y^2$.

Q.4. Find the volume of the region that lies inside both the sphere $x^2+y^2+z^2=4$ and the cylinder $x^2+y^2=1$.

Q.5 Evaluate

$$\iiint_Q \left(x^2+y^2+z^2\right) dV,$$

where Q is the region that lies inside the sphere $x^2+y^2+z^2=4$ and outside the sphere $x^2+y^2+z^2=1$.

Department of Mathematics

College of Science King Saud University

M-203 (Diff. and Integral Calculus)

II Mid-term Exam. (I semester 1446 / 2024/2025)

Time: 90 Mts. Max. Marks: 25

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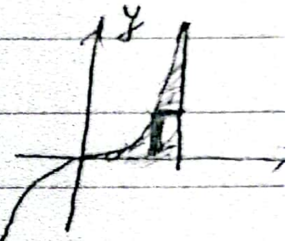
Solutions to the Questions.

Q#1) Evaluate the double Integral $\int_0^1\int_{\sqrt[3]{y}}^1 \frac{1}{1+x^4}\,dx\,dy$

Soln. We reverse the order of the Integral:



$$\int_0^1\int_0^{x^3} \frac{1}{1+x^4}\,dy\,dx$$

$$= \frac{1}{4}\int_0^1 \frac{4x^3}{1+x^4}\,dx = \frac{1}{4}\ln(1+x^4)\Big]_0^1 = \frac{1}{4}\ln(2).$$

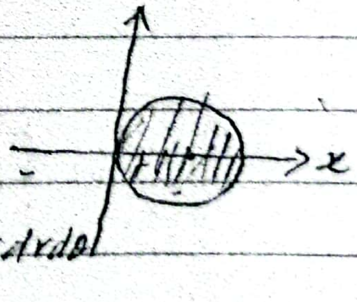
Q#2) Evaluate the double Integral: $\iint_R 4+\sqrt{x^2+y^2}\,dA$

where R is the region inside the circle $x^2+y^2=2x$.

Soln. We change it to polar Coordinates.

we have $x^2+y^2=2x$

or, $r^2 = 2r\cos\theta$



$$\iint_R 4+\sqrt{x^2+y^2}\,dA = \int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta}(4+r)\,r\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\left[\cancel{4}^{2}\frac{r^2}{\cancel{2}} + \frac{r^3}{3}\right]_0^{2\cos\theta} d\theta = \int_{-\pi/2}^{\pi/2}\left(2\cdot 2^2\cos^2\theta + \frac{8}{3}\cos^3\theta\right)d\theta$$

$$= 16\int_0^{\pi/2}\cos^2\theta\,d\theta + \frac{16}{3}\int_0^{\pi/2}\cos^3\theta\,d\theta$$

$$= \cancel{16}^{8}\int_0^{\pi/2}\frac{1+\cos 2\theta}{\cancel{2}}\,d\theta + \frac{16}{3}\int_0^{\pi/2}(1-\sin^2\theta)\cos\theta\,d\theta$$

$$= 8\int_0^{\pi/2} d\theta + 8\int_0^{\pi/2} \cos 2\theta\, d\theta + \frac{16}{3}\left(\frac{2}{3}\right)$$

$$= \cancel{8}^{4}\frac{\pi}{\cancel{2}} + 8\left[\frac{\sin 2\theta}{2}\right]_0^{\pi/2} + \frac{32}{9}$$

$$= 4\pi + \frac{32}{9}$$

Q#3) Find the surface area of the portion of the cone $z=\sqrt{x^2+y^2}$ that lies inside the paraboloid $2z = x^2+y^2$

Soln. We have $z=\sqrt{x^2+y^2} = g(x,y)$

$$g_x = \frac{1(2x)}{2\sqrt{x^2+y^2}} \quad \text{and} \quad g_y = \frac{1(2y)}{2\sqrt{x^2+y^2}}$$

$$\therefore S.A. = \iint_R \sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}\, dA$$

$$= \iint_R \sqrt{\frac{x^2+y^2+x^2+y^2}{x^2+y^2}}\, dA = \iint_R \sqrt{\frac{2(x^2+y^2)}{(x^2+y^2)}}\, dA$$

Also, we have

$2z = x^2+y^2 = r^2$

$z = \sqrt{x^2+y^2} = r$

$\therefore \frac{1}{2}r^2 = r$

or, $r^2 = 2r$

or, $r^2 - 2r = 0$

or, $r(r-2) = 0$

$\therefore r=0$ or $r=2$

$$= \sqrt{2}\iint_R r\,dr\,d\theta = \sqrt{2}\int_0^{2\pi}\int_0^2 r\,dr\,d\theta$$

$$= \sqrt{2}\; 2\pi\left[\frac{r^2}{2}\right]_0^2 = \sqrt{2}\cdot 2\pi(2)$$

$$= 4\pi\sqrt{2}$$

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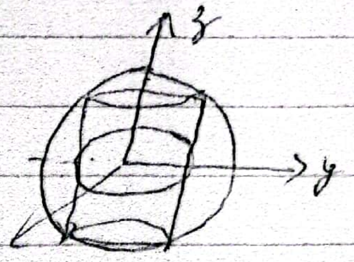
Q#4) Find the volume of the region that lies inside both the sphere $x^2+y^2+z^2=4$ and the cylinder $x^2+y^2=1$

Soln. The volume $V = \int_0^{2\pi}\int_0^1\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

$$= \int_0^{2\pi}\int_0^1 r\left[2\sqrt{4-r^2}\right]dr\,d\theta$$



$$= 2\pi\int_0^1\left(2r\sqrt{4-r^2}\right)dr$$

$$= 2\pi\left(\frac{16}{3}-2\sqrt{3}\right) \approx 11.7447$$

Put $4-r^2=u \Rightarrow$

$-2r\,dr = du$

$-\int\sqrt{u}\,du$

$= -\frac{2}{3}u^{3/2}$

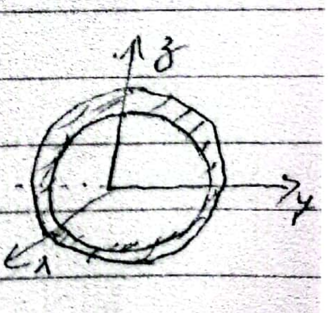
$= -\frac{2}{3}\left(4-r^2\right)^{3/2}\Big]_0^1$

$= -\frac{2}{3}\left(3^{3/2}-2^{2\cdot 3/2}\right)$

$= -\frac{2}{3}\left(3^{3/2}-2^3\right)$

$= \frac{16}{3}-2\sqrt{3}$

Q#5) Evaluate $\iiint_Q (x^2+y^2+z^2)\,dV$, where Q is the region that lies inside the sphere $x^2+y^2+z^2=4$ and outside the sphere $x^2+y^2+z^2=1$.



Soln: $\int_0^{2\pi}\int_0^{\pi}\int_1^2 \rho^2\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$

$$= \int_0^{2\pi}\int_0^{\pi}\left[\frac{\rho^5}{5}\right]_1^2 \sin\varphi\,d\varphi\,d\theta$$

$$= \frac{31}{5}\times 2\pi\left[-\cos\varphi\right]_0^{\pi} = \frac{62}{5}\pi(2) = \underline{\underline{\frac{124}{5}\pi}}$$