King Saud University

Department Of Mathematics

M-203

(Differential and Integral Calculus)

First Mid-Term Examination

(I-Semester 1435/36)

Max. Marks: 25 | Time: 90 Minutes

Q. No: 1 Determine whether or not the sequence

$$\left\{ n\left(\sqrt{n^2+2}-\sqrt{n^2-3}\right)\right\}_{n=\cancel{1}\,2}^{\infty}$$

converges, and if it converges find its limit.............................[4]

Q. No: 2 Determine whether the following infinite series converges or diverges. If it converges, find its sum

$$\sum_{n=2}^{\infty}\frac{1}{n^2-1}$$ ....[5]

Q. No: 3 Test whether the following series is absolutely convergent, conditionally convergent, or divergent

$$\sum_{n=1}^{\infty}(-1)^n\left(1+\frac{1}{n^2}\right)$$ ....[4]

Q. No: 4 Find the interval of convergence and radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{4^n}{\sqrt{n}}x^n$ .... [6]

Q. No: 5 Find Maclaurin series for $f(x)=\sin x$ and use the first three non-zero terms to approximate

$$\int_0^1 x\sin(x^2)\,dx$$ ....[6].

Q:1 $a_n = n\left(\sqrt{n^2+2}-\sqrt{n^2-3}\right) \times \dfrac{\left(\sqrt{n^2+2}+\sqrt{n^2-3}\right)}{\left(\sqrt{n^2+2}+\sqrt{n^2-3}\right)}$ 2

$$= n \, \frac{(n^2+2)-(n^2-3)}{\sqrt{n^2+2}+\sqrt{n^2-3}} = n \, \frac{5}{\sqrt{n^2+2}+\sqrt{n^2-3}}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5}{\sqrt{1+2/n^2}+\sqrt{1-3/n^2}} = \frac{5}{2}$$ 2

Q:2 $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)} = \sum_{n=2}^{\infty}\left[\frac{1}{2(n-1)} - \frac{1}{2(n+1)}\right]$ 2

$$S_n = \left[\frac{1}{2(1)} - \cancel{\frac{1}{2(3)}}\right] + \left[\frac{1}{2(2)} - \cancel{\frac{1}{2(4)}}\right] + \left[\cancel{\frac{1}{2(3)}} + \frac{1}{2(5)}\right]$$
$$+ \left[\cancel{\frac{1}{2(4)}} - \frac{1}{2(6)}\right] + \cdots \left[\frac{1}{2(n-1)} - \frac{1}{2(n+1)}\right]$$ 2

$$= \frac{1}{2} + \frac{1}{4} + \left[\frac{1}{2(n-1)} - \frac{1}{2(n+1)}\right] \longrightarrow \frac{3}{4}.$$ 1

Q:3 First a.c. $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n^2}\right)$

use nth term test $\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1+\frac{1}{n^2}\right) = 1$

$\Rightarrow$ d'gt

Now using AST (ii) $\displaystyle\lim_{n\to\infty} a_n = 1 \neq 0$ 3

So given series is d'gt. 1

Q:4 $\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{4^{n+1}x^{n+1}}{\sqrt{n+1}} \times \frac{\sqrt{n}}{4^n x^n}\right|$

$= \lim_{n\to\infty} \frac{4\sqrt{n}}{\sqrt{n+1}}|x| = 4|x|$ 2

c'gt $\Rightarrow 4|x|<1 \Rightarrow |x|<\frac{1}{4} \Rightarrow -\frac{1}{4}<x<\frac{1}{4}$

Check c'gence at $x=-\frac{1}{4}$

$\sum_{n=1}^{\infty} \frac{4^n(-1/4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

By AST it is c'gt 1

Check c'gence at $x = 1/4$

$\sum_{n=1}^{\infty} \frac{4^n(1/4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

d'gt p-series 1

Interval of c'gence $-1/4 \le x < 1/4$ 1

Radius of c'gence $\rho = \frac{1/4+1/4}{2} = 1/4$ 1

Q:5 $f(x) = \sin x \Rightarrow f(0) = 0$

$f'(x) = \cos x \Rightarrow f'(0) = 1$

$f''(x) = -\sin x \Rightarrow f''(0) = 0$

$f'''(x) = -\cos x \Rightarrow f'''(0) = -1$

⋮

Maclaurin Series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\cdots = \sin x$ 3

$\int_0^1 x\left[x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots\right]dx \approx \int_0^1 \left[x^3 - \frac{x^7}{3!} + \frac{x^{10}}{5!} - \cdots\right]dx$

$= \left.\frac{x^4}{4} - \frac{x^8}{48} + \frac{x^{11}}{1320}\right]_0^1 = \frac{1}{4} - \frac{1}{48} + \frac{1}{1320} \approx 0.25 - 0.20833 + 0.000757$

≈ 0.0424 3