KING SAUD UNIVERSITY
College of Sciences
Department of Mathematics

Final examination/ Summer semester / 1427/1428

Math 203, Time: 3 hours

- **Question 1.** [Marks: 3+4+5]

a) Check the convergence or the divergence of the series : $\sum_{n=0}^{\infty}[\frac{1}{2^n}+(-\frac{3}{4})^n]$. Find the sum if it is convergent.

b) Determine whether the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{lnn}$ is absolutely convergent, conditionally convergent or divergent.

c) Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n3^n}(x-1)^n$.

- **Question 2.** [Marks: 4+4+4]

a) Find the Maclaurin series of the function $Sinx$ and use its first three non-zero terms to approximate the improper integral $\int_{-1}^{1}\frac{Sinx}{x}dx$.

b) Evaluate the integral $\int_{1}^{e}\int_{\frac{1}{e}}^{\frac{1}{y}}Cos(x-lnx)dxdy$.

c) Find the surface area of the solid bounded above by the surface $z = 9-x^2-y^2$ and below by the xy-plane.

- **Question 3.** [Marks: 4+4+4]

a) A solid is bounded by the paraboloid $z=x^2+y^2$, the cylinder $x^2+y^2=4$ and the xy-plane. Find its centroid.

b) Using spherical coordinates, evaluate the integral

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}}z^2\sqrt{x^2+y^2+z^2}dzdydx.$$

c) Show that the following line integral is independent of path, and find its value :

$$\int_{(-4,3)}^{(5,2)}(y^2+2xy)dx+(x^2+2xy)dy.$$

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- **Question 4.** [Marks: 4+5+5]

a) Use Green's theorem to evaluate the line integral $\oint_C y^3 dx + (x^3 + 3xy^2) dy$, where C is the path from $(0,0)$ to $(1,1)$ along the graph of $y = x^3$ and from $(1,1)$ to $(0,0)$ along the graph of $y = x$.

b) Use the divergence theorem to find $\iint_S \vec{F} \,.\, \vec{n} \; ds$ if $\vec{F}\,(x,y,z) = (x^2 + Sinz)\,\vec{i} + (xy + Cosz)\,\vec{j} + e^y\,\vec{k}$, S is the surface of the region bounded by the cylinder $x^2 + y^2 = 4$, the plane $x + z = 6$ and the xy-plane.

c) Use Stokes's theorem to evaluate $\oint_C \vec{F} \,.d\, \vec{r}$, where $\vec{F} = 2z\,\vec{i} + x\,\vec{j} + y^2\,\vec{k}$ and S is the surface of the paraboloid $z = 4 - x^2 - y^2$ and C is the trace of S in the xy-plane.

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