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| King Saud University<br>Department Of Mathematics.<br>College of Sciences. | Final Exam M.203<br>Differential and integral<br>Calculus. | Summer semester<br>We. 14/11/1437H.<br>Time: 3 hours. |

**Question 1.** $[2+2+4]$

a) Determine whether if the following of series are convergent or divergent.

$$\sum_{n=1}^{\infty}(-1)^n \frac{3^n}{n^2} \quad , \quad \sum_{n=1}^{\infty} \frac{\sin(3n)+1}{2^n}$$

b) Find only the first four terms of the **Taylor** series of the function

$$f(x)=\ln(x+3) \quad , c=-1$$

**Question 2.** $[4+5]$

a) Find the interval and radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n\,2^n}.$$

b) Find the volume of the solid bounded by the cone $z=\sqrt{x^2+y^2}$ and the paraboloid $z=2-x^2-y^2$.

**Question 3.** $[5+5]$

a) Evaluate the following integral : $\iiint\limits_Q z^2 dV$ , where $Q$ is the part of the sphere $x^2+y^2+z^2 \le 1$ that lies in the first octant.

b) Show that the line integral is independent of path , and find its value.

$$\int_{(\pi,-1)}^{(\frac{\pi}{2},2)} (-2y^3 \sin x)dx+(6y^2 \cos x+5)dy$$

**Question 4.** $[5]$

Let $Q$ be the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ , and $z = 3$ . Let $S$ be the surface of $Q$ .For the vector field

$\vec{F}(x,y,z) = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ , use the Divergence Theorem to evaluate the integral $\iint_S \vec{F}.\vec{n}\,ds$ .

**Question 5.** $[4+4]$

Let the force $\vec{F}$ defined by $\vec{F}(x,y,z) = -4y\,\vec{i} + 2z\,\vec{j} + 3x\,\vec{k}$ and let $S$ be the portion of the paraboloid $z = 10 - x^2 - y^2$ above the plane $z = 1$ .Verify Stokes' Theorem.